same form, its factors can be found. However, the limits are smaller than those of Tchebychef, and x, y may be either integers, or fractions with denominator 2. These determinants are: 5, 21, 77, 165, 285, 357, 437, 957, 1085, 1365, 2397, 2805, 4485, 7917, 8645, 26565. In each case $T^2-D=4$. If $ax^2-by^2=m$ is possible where $ab=D$, there is a representation with

$$\left(\frac{m}{a}\right)^{1/2} \leqq x \leqq \left(\frac{(T+2)m}{4a}\right)^{1/2}.$$

10. In his paper Professor Roberts suggested that the graduate attitude be introduced into elementary courses, and he made a plea for more vital teaching.

11. Consider the equilateral triangle ABC. With A as center and the altitude of the triangle as radius, swing an arc $A'B''$ from the center A' of BC to the point B'' on the side AC. Let S denote the area $A'B''C$ and let P be a point of S. Form a triangle ABP and construct two lines $AR_i(i=1, 2)$ and PG of length $(\overline{AB}\sqrt{3})/2$ extending from the vertices A, P to their opposite sides BP, AB, where angle BGP is acute. If the points of intersection are denoted by $Q_i(i=1, 2)$, then Q_i are defined to be the transforms of the point P.

Professor Hinrichsen obtained the equations of transformation and studied the transforms of certain simple families of curves. Then he considered the transformation from Q_1 to Q_2. He found this transformation to be similar to a reflection in a straight line. It is one-to-one except along the straight line segment AB'' which is carried into the point A.

12. In this paper Professor Holl gave a simple geometric representation of moments and products of inertia of a plane area with the aid of a circle instead of the ellipse. He represented the scalar invariants as geometric invariants, and made extensions to curvatures of normal sections of a surface, also to stresses, strains and moments in elastic theory.

13. In this paper Professor McKelvey presents a detailed schedule of lessons covering approximately the first two weeks' instruction on the Definite Integral.

15. Let successive sets of n observations each be made upon a variable which is subject to a constant law of error and denote by x, y, z, $x \leqq y \leqq z$, the pth, qth and rth measurements of the sets. Doctor Craig derived the correlation function $V=\phi(x, y, z)$ and showed that the regression surfaces are planes. He proved that the correlation between the pth and rth measurements within one set of observations is equal to that between the pth measurement of one set and the rth measurement of a second set, the two sets having identical qth measurements. The correlation function $W=\psi(u, v)$ of $u=z-y$ and $v=y-x$ was also given. By assigning appropriate values to p, q, r, the correlation between various averages is readily determined.

J. F. REILLY, *Secretary-Treasurer*

THE APRIL MEETING OF THE ROCKY MOUNTAIN SECTION

The sixteenth regular meeting of the Rocky Mountain Section of the Mathematical Association of America was held at the University of Wyoming, Laramie, Wyoming, on Friday and Saturday, April 15 and 16, 1932. There were three sessions, Professor O. H. Rechard, chairman of the Section, presiding at each.

The attendance was thirty-five, including the following thirteen members of the Association: Jack Britton, Pauline F. Folk, G. W. Gorrell, C. A. Hutchinson, M. H. Ingraham, A. J. Kempner, Claribel Kendall, A. J. Lewis, S. L. Macdonald, A. S. McMaster, O. H. Rechard.

At the business meeting the following officers were elected for the ensuing year: Chairman, A. G. Clark, Colorado Agricultural College; Vice-Chairman, W. V. Lovitt, Colorado College; Secretary, A. J. Lewis, University of Denver.

Members of the Association and friends were guests of the University of Wyoming at a dinner on the evening of April 15. The principal speakers at the dinner were President A. G. Crane, of the University of Wyoming, and Professor S. L. Macdonald, of the Colorado Agricultural College. The Section was fortunate in having Professor M. H. Ingraham, of the University of Wisconsin, present as guest speaker.

The following seven papers were read:

1. "Operational calculus" by Professor C. A. Hutchinson, University of Colorado.

2. "Contributions of mathematics to life insurance" by Professor G. W. Gorrell, University of Denver.

3. "The Baire classification of functions" by Professor O. H. Rechard, University of Wyoming.

4. "The development of the postulational method in mathematics" by Professor M. H. Ingraham, University of Wisconsin.

5. "The teaching of calculus" by Professor S. L. Macdonald, Colorado Agricultural College.

6. "Geometric progressions" by Professor A. J. Kempner, University of Colorado.

7. "Controversial topics in mathematical logic" by Professor M. H. Ingraham, University of Wisconsin.

Abstracts of the papers follow, the numbers corresponding to the numbers in the list of titles:

1. This paper is expository in character, and presents the salient features of the operational calculus, as applied to the solution of problems in electrical engineering.

2. This paper gives a brief history of the problem of life insurance and shows the role mathematics has played in its development.

3. In this paper, Professor Rechard presents the Baire method of classifying functions as given by Baire in his dissertation "*Sur les functions de variables*

réelles" published in "Annali di Matematica Pura et Applicata" in 1899. Functions of classes one, two, and three are exhibited and the classical proofs are given that the classification can be correct to any number α of the first or second class, but is not exhaustive. Comparison is made between the Baire, Young, and Sierpinski methods of classification. Some of the questions which need to be answered before a necessary and sufficient condition can be found for a function to be of class two, Baire, are suggested.

4. This paper discusses the historical development of the postulational method, and the major characteristics and uses of this method. Especial attention is paid to the use of the method for generalization, in which case the postulates should be non-categorical, and for establishing isomorphisms in which case they should be categorical. As illustrations, the postulates for a field, for Euclidean geometry, and Huntington postulates for an arithmetic mean are used. It is pointed out that at least logic is generally assumed as a background for sets of mathematical postulates.

5. It is the belief of the writer that in an elementary course in calculus, definitions, principles and descriptive matter should be reduced to a minimum consistent with clearness and rigor. It is maintained in this paper that most text books are at fault in this particular. The paper maintains that the derivative is not a rate, it is not a slope. The derivative is the limit of a ratio and is an entirely abstract concept. Rate and slope are merely properties of the derivative. By making clear that in certain cases a distinction is necessary between the limit of a ratio and the ratio of the limits the writer holds that the definition of the derivative may be clarified, which is seldom done by text book writers.

7. This paper discusses some of the current attempts to examine and explain the relation of logic to mathematics. In particular three schools of thought are considered. 1) The school led by Russell which attempts to define all mathematics in logical terms. 2) The school led by Brouwer which makes mathematics prior to logic and places stringent limitations on the use of classical logic. 3) The school led by Hilbert which is interested in the formal structure of mathematics and the questions of formal consistency, and studies mathematics as a set of marks on paper which are made in accordance with certain rules. The attempt is made to give a sympathetic discussion of each of these three points of view.

A. J. LEWIS, *Secretary*

ERNEST JULIUS WILCZYNSKI

Ernest Julius Wilczynski was born in Hamburg, Germany, on November 13, 1876, and died in Denver, Colorado, on September 14, 1932, after a lingering illness of about ten years. With respect to his original contributions to existing mathematical knowledge, his influence on the development of mathematical institutions in the United States, his interest in the promotion of good teaching,

and his heroic example of how a great man can meet and endure physical disability, he set a shining example and proved himself to be a leader of whom we may well be proud.

Wilczynski's early life and education can be briefly sketched. After he had gone to elementary school but a short time in Hamburg, his family migrated to the United States and settled in Chicago. Here Wilczynski graduated from the North Division High School. Then, with the assistance of an uncle, he returned to Germany for the purpose of studying at the University of Berlin. His interest lay in the fields of mathematics, physics and mathematical astronomy, and he specialized in the latter. He received the degree of doctor of philosophy from the University of Berlin in 1897, while still in his twenty-first year, the subject of his thesis being "Hydrodynamische Untersuchungen mit Anwendung auf die Theorie der Sonnenrotation."

Wilczynski returned to the United States after receiving his degree, and for a year was a computer in the Office of the Nautical Almanac at Washington. Then in 1898 he secured a post as instructor in mathematics at the University of California, and thereafter his advancement was rapid. At California he served until 1907 as instructor, assistant professor, and associate professor, with an interruption from 1903 to 1905 when he was in Europe as research assistant and associate of the Carnegie Institution of Washington. He was associate professor at the University of Illinois from 1907 to 1910, and at the University of Chicago from 1910 to 1914. He became professor of mathematics at Chicago in 1914, and was made professor emeritus in 1926 when it was clear after three years of ill health that he would never be able to resume his active duties.

In spite of the fact that his greatest reputation was made as a geometer, Wilczynski began his scientific career as a mathematical astronomer and made notable contributions in this field. Then his interest turned to projective differential geometry. He developed a new method in this domain, and his energy in applying it soon won for him wide recognition as the creator of a new mathematical discipline. Toward the end of his career he was contributing to the theory of functions of a complex variable.

It must not be thought that Wilczynski became a great research investigator at the expense of interest or proficiency in teaching. In fact he was a very clear and effective teacher. He had a beautiful English style, and his lectures both to graduate and undergraduate students were models of elegant mathematical exposition. His interest in instruction at the undergraduate level is exemplified by the two texts that he wrote for freshmen, one a trigonometry and the other a college algebra. In the trigonometry, especially, his fondness for the heuristic method of presentation found ample scope. It was a tenet of his that the first prerequisite for effective instruction was that the teacher should win the confidence of his students and should make them come to feel thoroughly at ease in his class room. After all, perhaps the most eloquent comment on his qualities as a teacher is the fact that his advanced courses were most popular with graduate students of mature years who were themselves experienced teachers, coupled

with the fact that Wilczynski directed the theses of twenty-five candidates for the doctor's degree, besides innumerable master's theses.

Wilczynski was interested in the affairs of several scientific organizations and was honored by them from time to time in various ways. He was at one time vice-president of the American Mathematical Society, and a member of the council of the Mathematical Association of America. He was a lecturer at the New Haven Colloquium, and associate editor of the Transactions of the Society. In 1909 he won a prize of the Royal Belgian Academy of Sciences, and in 1919 was elected a member of the National Academy of Sciences at Washington.

Wilczynski is survived by his wife and three daughters. Their loss is shared by his former students and colleagues, as well as by many other mathematicians who have come under his influence. The memory of his genial and friendly personality, his power as a teacher, and his enthusiasm for mathematics, the material evidence of his seventy-seven books and papers published in four languages, and nineteen periodicals, together with the monument of a new domain of geometry so largely created by him—these things constitute a legacy in which all of us may find a great wealth of inspiration.

ERNEST P. LANE

COLLEGIATE MATHEMATICS NEEDED IN THE SOCIAL SCIENCES[1]

The background and purpose of this report are indicated by parts of the letter which Professor E. B. Wilson, President of the Social Science Research Council in June, 1930, sent to Professor Tolley informing him of the appointment of the committee and its duties.

"You are of course aware of the request by the Advisory Committee on Social and Economic Research in Agriculture for the appointment of a committee of this Council on mathematics to determine what mathematics should be taught to students of the social sciences . . . The Committee on Problems and Policy in thinking the matter over decided that the time was not ripe for the appointment of a national committee but decided to appoint a sub-committee of the Advisory Committee on Agriculture which . . . should make a preliminary report on the subject. The only way I see of making progress is for the social scientists to know what they want, to explain to the mathematicians the

[1] This report was prepared for the Social Science Research Council by a Committee consisting of H. R. Tolley, Director of the Giannini Foundation of Agricultural Economics, University of California; F. L. Griffin, Professor of Mathematics, Reed College; Holbrook Working, Economist of the Food Research Institute, Leland Stanford University; Charles H. Titus, Professor of Political Science, University of California at Los Angeles; and Mordecai Ezekiel, Assistant Chief Economist, the Federal Farm Board; and submitted to the Advisory Committee on Social and Economic Research in Agriculture. The report was presented and discussed (See this MONTHLY, vol. 39, 1932, p. 503) at the meeting of the Mathematical Association in Los Angeles in September, 1932.

types of problem they have to solve and the kind of mathematics they do use so that the mathematicians on the committee may orient their thinking on a basis of reality."

Although the appointment of this Committee was the outgrowth of discussions in the Advisory Committee on Economic and Social Research in Agriculture concerning graduate training for students in Agricultural Economics and Rural Sociology, the Committee is of the opinion that the needs of students in Agricultural Economics and Rural Sociology for training in collegiate mathematics are not essentially different from the needs of students who specialize in other branches of economics and sociology, and that all students of the social sciences would find some work in collegiate mathematics very helpful. Hence the needs of all students of the social sciences, in so far as they are known to the committee, have been considered. Training in mathematics will be most helpful if obtained prior to courses in statistics and other subjects in which mathematical terms, concepts, and methods are or may be used. Hence the committee feels that students should plan, wherever possible, to obtain most if not all of their college training in mathematics in the early years of undergraduate work.

Since the committee was requested "to determine what mathematics should be taught to students of the social sciences," they considered the desirability of recommending certain courses to be required of all students of the social sciences or possibly alternative courses to be required of different classes of students. However, after careful consideration, the committee reached the conclusion that such recommendations could be made only in very restricted form because for some social science students there is a fair question whether substantial direct benefit would be realized from any college mathematics whatever. On the other hand some few students might profitably take even more mathematics than is involved in an ordinary undergraduate mathematics major. There is need in nearly every institution doing graduate teaching or research in the social sciences for at least one staff member actively engaged in social science work whose mathematical and statistical training is the equivalent of what might be required for a Doctor of Philosophy degree in mathematics and statistics.

In framing any general requirement for mathematics courses, it must be remembered that in the early stages of the undergraduate career—when the training in mathematics should be obtained—it is rarely possible for the student to judge whether his subsequent study and work will call for much or little mathematics. The committee believes, however, that a knowledge of the phases of collegiate mathematics listed below would be helpful and useful to a large proportion of the students in economics, and that many students in other social sciences would find it worth while to obtain the knowledge and training that can be acquired through the study of these phases of mathematics, providing it can be done without taking too much time from the study of other subjects. The topics suggested are listed here by their usual mathematical names. The list is followed by a brief comment upon some of the social science objectives sought through the study of the topics. There is no significance in the order or

grouping; various groups might well be separated and rearranged in laying out a systematic course.

(1) *Logarithms.* Common logarithms; numerical computation; compound interest and annuities; Napierian logarithms.

(2) *Graphs* (as a tool in the study of tabulated data). Plotting on ordinary and logarithmic papers; measurement of slopes and areas; graphical determination of maxima and minima; mean values; representation of three-variable relationships by means of three-dimensional diagrams and contours.

(3) *Interpolation.* By reading ordinary or logarithmic graphs; by proportional parts; by successive differences.

(4) *Equations and forms of curves.* Basic mathematical functions, their graphs, and the characteristic properties of the variations represented; straight lines; parabolic and hyperbolic curves; exponential and logarithmic curves; logistic curves; sine and cosine curves.

(5) *Probability.* Combinations, binomial theorem, elementary probability; the normal probability curve—its form, table, and equation; probability and frequency distributions; probability and time series.

(6) *Elements of differential and integral calculus.* The significance of a derivative as a limit, as a rate or slope, as a frequency, etc., the differential as an approximate increment; formulas for differentiating elementary mathematical functions, as u^n, a^u, $\log u$, $\sin u$, $\cos u$, uv, u/v; partial differentiation; procedure for determining maximum and minimum values; integration as the reverse of differentiation; relation between integration and summation; multiple integration.

(7) *Curve fitting* (mathematical principles). Plotting tabulated values, their logarithms, reciprocals, etc., to ascertain whether the tabulated points lie on a curve of simple form; determination of the curve through such points, or selected points within the scatter-diagram or chart; the method of least squares or other methods for obtaining the constants of curves of best fit.

In organizing a course or courses embracing these topics, *illustrative material from the social sciences should be used freely, and the concepts and processes should be presented in such a manner as to make clear their application in the social sciences.* Special effort should be made to give the student an insight into the significance of mathematical ideas and processes, and to their applicability or non-applicability in the study of problems in the social sciences; to acquaint him with mathematical expression and reasoning as a mode of thought; and to cultivate a critical attitude in scrutinizing assumptions and the uses made of assumptions.

In suggesting topics to be covered in a specially organized course or courses the committee has attempted to select those topics having the greatest significance and earliest utility for students in the social sciences. The work with logarithms would not only facilitate numerical calculations in general, but would also enable students to understand financial mathematics as in funding debts, etc., and serve as a basis for understanding various exponential curves, prob-

ability, etc.;—as well as logarithmic plotting systems connected with studies of percentage rates of growth (of population, etc.). Graphical representation of tabulated data and of mathematical functions besides being basic for statistical analysis, facilitates the understanding of economic variables and relationships (e.g. supply and demand curves, and records of prices; rates of change and cumulative effects; average of a changing quantity like the national debt or volume of unemployment, or mortality rates, etc.). Interpolation problems are well-nigh universal.

The study of the equations and forms of curves should make the student sufficiently familiar with the forms and characteristics of the simpler curves to enable him to recognize in many cases the type of curve suitable for fitting to a given body of data, and to appreciate the principal implications involved in the use of a curve. The student should become familiar with types of cyclical and undulatory movements, with common approximations to trends and growth curves (for prices, populations, political institutions, etc.), and with varieties of theoretical demand curves, unit-cost curves, etc. The work in probability, besides affording a basis for statistical study in curve fitting and studies of distributions and variability (e.g., of scores in tests, of incomes, divorces at various ages, degrees of radicalism in political opinion, etc.), would assist in understanding contingent values in finance and in risks generally.

The elementary work in calculus would enable students to understand the relations between marginal and total utility, rates and cumulative growth, frequency and cumulative frequency, etc., the conditions for a maximum (e.g., profit); the change in one quantity due to slight changes in several others upon which it depends (say, the size of the crop from a certain field, as affected by slight changes in rainfall, fertilizer input, temperature, cultivation, etc.); concepts like the elasticity of demand; partial and multiple correlation procedures, curve fitting with normal equations, etc. It would also enable students to follow many of the discussions in the literature where the symbols and concepts of differentiation and integration are employed.

The present organization and arrangement of the courses in many colleges and universities is such that 15 to 20 semester hours of mathematics must be taken in order to obtain a knowledge of the above topics. Further, the courses are usually taught without any reference to the application of the mathematical concepts and terms to problems in the social sciences. Under these circumstances comparatively few students in any of the social sciences feel justified in spending the time required to take these courses. In the judgment of the committee, the advantage of mathematical training to the majority of undergraduate majors in economics (the social science discipline in which mathematical training now appears to be most useful) is not sufficient to warrant the displacement of 15 to 20 semester hours of other work, wisely chosen, by that amount of mathematics lacking definite correlation with social science.

The committee is of the opinion that, if the mathematics courses be properly organized, the topics listed above can be taught to students who have had only

ordinary high school mathematics in a form readily grasped by them and full of significance to them; also, that this can be done, with opportunity for ample practice, in a three-hour course running through two, or at most, three semesters. Students who desire even more practice in manipulation could be permitted to register for an extra "hour" or "unit" of such work paralleling the course.

As will be discussed presently, some students specializing in social science will need a considerably more extensive knowledge of mathematics than is obtainable from the foregoing course. They can, however, use this first course as a foundation for more advanced courses, and go as far as their special needs dictate. From the experience of various colleges with higher courses based upon a unified first course of somewhat similar character, it appears likely that it would be possible to cover by the end of the second year all the topics and technique of the ordinary calculus that are of importance to social scientists, and also to introduce some ideas usually postponed to a course in advanced calculus.

The training in such mathematics courses would help the student to acquire facility in reducing to definite terms the concepts and relationships encountered in the courses in the social sciences and to do clear and logical thinking and reasoning.

This opinion is based on the experience of members of the committee in teaching mathematics, as such, and in teaching advanced courses in statistics to students who required to be taught a substantial amount of this mathematics along with the statistics, and on reports of experience with courses designed for this purpose in a number of colleges and universities.

In view of all these considerations, the committee feels that if a specially organized course such as suggested above were available, many students in the social sciences would find it worth their while to take it. The committee has no hesitancy in recommending that this amount of mathematics should, if available, be fairly generally required of students specializing in economics and in some others of the social sciences. However, since only economists, political scientists, and mathematicians were represented on the committee, we feel that representatives of other social sciences should be asked to give further consideration to this point.

Such training would prove of immediate benefit to students in subsequent courses in statistics and for economics students in finance and in economic theory, if the subsequent courses presupposed a knowledge of mathematics and took advantage of that preliminary training. A substantial percentage of social science students would find further direct benefit from such a course amply repaying them for the time spent. Even those students who failed to realize direct benefit from the mathematical training in subsequent study and work would usually find the cultural and general educational value of such a course equal to that which might have been derived from an equivalent amount of time devoted to alternative subjects not directly related to their principal line of interest.

It should be said that the success of any such course in meeting the need we

envisage must depend upon the knowledge and ability of the instructor as much as upon the choice of topics for instruction. The instructor must possess the understanding of mathematics necessary to carry the students beyond the mechanics of formal mathematical processes to an understanding of the concepts involved, and must also possess the knowledge of social sciences to enable him not only to provide illustrations of uses of the mathematical methods in the social sciences, as well as in other sciences, but also to relate the mathematical concepts to concepts employed in the social sciences.

Preparation for Courses in Statistics. Mathematics courses covering the above topics would be very helpful as a preliminary to a first course in statistics. The second encounter with the mathematical terms and concepts when taking courses in statistics will serve to give a more comprehending grasp than would be possible with a single encounter. Moreover, with such mathematical preparation the student in the statistics course would be released from spending time in learning the mathematical topics *de novo*, and from distractions of attention incident upon such digressions. The statistics course could then utilize the usual amount of time to carry the student much farther in the knowledge of statistical methods, their possibilities and limitations; or the customary ground could be covered in a much shorter time.

Preparation for the Study of Economic Theory. Training in mathematics is useful to those who take undergraduate courses in economic theory in that it facilitates rapid and sound reasoning on many problems encountered in these courses. Many phases of economic theory, which seem quite involved and difficult to one who has not been trained to think in terms of equations, graphs, and small differences, can be stated quite simply in mathematical terms and are easily understood by one who is able to grasp the meaning and significance of these terms.

Despite these advantages few instructors in economic theory have felt justified in urging their students to take the courses in mathematics, as customarily offered (trigonometry, college algebra, analytical geometry, and calculus), which require some 15 to 20 semester hours of work. But the availability of briefer courses including mathematics of principal use in the study of economic theory and omitting unnecessary technique and complications would go far toward swinging the balance of judgment to the other side.

Although some of the topics that would be covered have little direct application in the study of those phases of economic theory usually covered in undergraduate courses, most students who specialize in economics take a course or courses in statistics as well as in economic theory, and the committee is of the opinion that such students would be amply repaid for the time spent in taking a mathematics course such as this outlined above.

Preparation for Research and Teaching in the Social Sciences. After obtaining the bachelors degree, a large majority of students who specialize in the social sciences now enter upon business careers or into other lines of work in which they find little direct use for the knowledge that can be gained in courses in

collegiate mathematics. The inclusion of mathematics in the course of study of such students must be justified on the basis of the cultural training and mental discipline obtained and on the basis of the usefulness of mathematical concepts and methods in other courses, such as economic theory, statistics and accounting.

With the student who expects to become a professional social scientist, however, the situation is very different. The student who expects to become an instructor or investigator will usually spend two or more years in graduate work to obtain additional training in his chosen field. In this time, he will endeavor to prepare himself to teach his subject to both undergraduate and graduate students, to carry on independent research, to make contributions to knowledge in his field, and to read and understand the contributions of his colleagues.

Students preparing to become instructors in economic theory or to do research in a field requiring a comprehensive knowledge of economic theory will find much more direct use for a knowledge of mathematics than will most of those whose collegiate training is limited to undergraduate study. In particular, it will make available the writings of mathematical economists and enable one to understand and evaluate the work of statistical analysts. Ordinarily, such students would do well to consider that it is essential to have mathematical equipment, at least equal to that which would be obtained in the brief courses suggested for undergraduates, and plan to obtain this equipment as part of their formal training, even if it must be done in connection with their graduate study.

The student who expects to become a statistical analyst should have a thorough grasp of statistical theory and its mathematical background. In the analysis of social statistics, acquaintance with the fundamentals of statistical theory is essential in order to devise and apply sound methods and to avoid faulty and fallacious analysis and interpretation of results. This implies the ability to read and understand the literature of statistical theory and method and to modify established methods or develop new ones in problems to which existing methods are not exactly applicable. A thorough acquaintance with differential and integral calculus and the theory of probability and an ability to handle abstract problems in these fields are necessary for work of this character. For a student who is looking forward to a career as a statistical analyst, training in mathematics considerably broader than can be obtained in the abbreviated course outlined above, will be very helpful.

Similarly those who expect to follow the writings of the mathematical economists or to make contributions to theory in those branches of economics in which mathematical concepts and methods are used must have a broad training in mathematics. Anthropologists who expect to deal with biometric problems, and psychologists who expect to work in psycho-physics need a thorough knowledge of calculus and the theory of probability.

On the other hand, among those who are now doing research and teaching in the social sciences, there are undoubtedly many who have found little or no

direct use for training in collegiate mathematics. Such is probably the case for most historians and for many sociologists, political scientists, and psychologists. It is possible that as further developments occur in the theory and research method of these disciplines, greater need for the general use of mathematical concepts and methods will become apparent, as has been the case in many of the natural and biological sciences and, more recently, in statistical analysis and economic theory. A social scientist should not be mystified by the appearance of a mathematical symbol or equation on the page of an article or in a text book, no matter in which of the disciplines he is primarily interested.

In view of all these considerations, the committee feels that a student who is planning to become a professional worker in any of the social science disciplines would be well repaid for the time and effort spent in taking six to nine semester hours of collegiate mathematics, provided the courses were so organized that the phases of mathematics mentioned in the first part of this report could be covered in that time. Some would find it worth while to take also a second-year course, or even the more extended courses in all these phases (even though they should require fifteen to twenty hours) if specially organized courses are not available.

The committee is cognizant of the fact that at present most of the graduate students in the social sciences have had no courses whatever in collegiate mathematics, but believe that many of these will find it worth while to make up their deficiencies even after beginning their graduate work. If abbreviated courses as outlined herein are available, those may be taken, along with more advanced courses which the student finds would be valuable to him. Some graduate students would, no doubt, be justified in adding a higher course, or in taking enough of the more extended courses ordinarily offered in mathematics to acquire an understanding of the differential and integral calculus and the mathematical theory of probability.

Mathematics "Requirements" in Social Science Curricula. Whether in any one of the social sciences, all students or only certain more or less restricted groups should be *required* to take courses in collegiate mathematics such as are suggested in this report, must depend to a considerable degree upon the degree of confidence with which need or lack of need for mathematical training can be predicted, upon the subjects which would be displaced if mathematics were required, and upon other circumstances which will differ as between disciplines within the social sciences and as between institutions.

The committee believes that at institutions where mathematics courses are available, in which students can obtain a knowledge of the fundamental concepts of differential and integral calculus and the elements of probability, with the necessary preliminary training, in a total of six to nine semester hours as in the special courses outlined, these courses should be made prerequisite to any course in statistics and to upper division courses in economic theory, and the courses in statistics and economic theory organized so as to take advantage of the knowledge gained in the mathematics courses. Perhaps certain other courses

in economics, and in the other disciplines as well, would on further consideration be included in the group for which the mathematics courses should be prerequisite.

In closing, the committee suggests (1) that further study of the problem with particular reference to the specific needs of the different social sciences and the coordination between courses in mathematics and the different social science disciplines is highly desirable, and (2) that interested institutions be encouraged to proceed with the development of courses in mathematics designed primarily to meet the needs of students in the social sciences.

SOME RELATIONS IN THE GEOMETRY OF THE TRIANGLE

By ALBERT A. BENNETT, Brown University

Let B_i $(i=1, 2, 3)$ be the vertices of a base triangle, with I as incenter, and C_i as points of contact of the inscribed circle, K, with radius r. Let R be the radius of the circumcircle, with a_i the tangent line at B_i. Let B_i' be the midpoint of the line segment B_iI. Let b_i be a side-line of the triangle $B_1B_2B_3$, b_i' a side-line of $B_1'B_2'B_3'$, and b_i'' the line parallel to b_i and b_i' through I. Let P_i be the midpoint of that arc B_jB_k of the circumcircle, not containing B_i, where here and hereafter i, j, k, denote distinct numbers of the set 1, 2, 3. Let p_i be the line P_jP_k.

Familiar or readily demonstrated theorems are the following:

1. Triangle $P_1P_2P_3$ is homothetic[1] with $C_1C_2C_3$.

2. P_i, I, B_i', B_i, are collinear. (Since P_i bisects arc P_jP_k and is therefore on B_iI.)

3. I is the orthocenter[2] of $P_1P_2P_3$, with B_i' as the foot of an altitude. (Since I and B_i are each equidistant from C_j and C_k, it follows that the line IB_i is perpendicular to C_jC_k and hence also to P_jP_k. Now b_j, b_k, b_j'', b_k'' form a rhombus with IB_i as one diagonal. Hence the midpoint B_i' of this diagonal is upon the line of the other diagonal. On the other hand the foot of the altitude is the point half way[3] between the orthocenter I and the second intersection B_i of the altitude with the circumcircle.)

4. p_i, b_j, b_k'', are concurrent, with p_i bisecting the angle between b_j and b_k''.

5. p_i, b_j', b_k', are concurrent, with p_i bisecting the angle between b_j' and b_k'.

6. P_i is equidistant[4] from I, B_j, B_k.

Hence we have readily the following new relations.

7. P_i is equidistant from the lines, b_j, b_k, b_i''.

8. P_i is equidistant from the lines, b_1', b_2', b_3'. Let this common distance be r_i'.

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[1] R. A. Johnson, *Modern Geometry* (1929), Page 194.

[2] Ibid, P. 194.

[3] Ibid, P. 163.

[4] Ibid, P. 185.

9. P_i is equidistant from the five lines, a_j, a_k, b_i, b_j'', b_k''. Let this common distance be r_i. That P_i is at the same distance r_i from b_i and b_j'' follows from the fact that it is on the angle bisector p_k. That P_i is at the same distance from b_i and a_j follows from the fact that P_i bisects arc B_jB_k, hence the line B_jP_i bisects the angle between the chord $B_jB_k=b_i$, and the tangent a_j at B_j.

10. $r+r_1+r_2+r_3=2R$.

Proof. The radius r' of the circle inscribed in $B_1'B_2'B_3'$ is half that of the circle inscribed in $B_1B_2B_3$, hence $2r'=r$. But $r_i'=r_i+r'$, and r_i' is the radius of the circle escribed to $B_1'B_2'B_3'$. Hence[1] $r_1'+r_2'+r_3'=4R'+r'$, where R' is the circumradius of $B_1'B_2'B_3'$, so that $4R'=2R$. Hence the theorem.

We may now add an intuitive proof of the following traditional theorem.[2]

In a triangle the outer common tangents to the excircles form a triangle whose incenter coincides with the circumcenter of the excenters, and the radius of whose incircle is equal to twice the circumradius plus the inradius of given triangle.

Proof. P_i is center of an escribed circle K_i' of $B_1'B_2'B_3'$, and the circumcircle of $B_1B_2B_3$ is the circumcircle of excenters. The radius of this centercircle is $2R'$. The external tangent a_i' to K_j' and K_k', other than b_i', is parallel to a_i and at a distance $r'=\frac{1}{2}r$ from it. Thus I is the incenter of the triangle a_1', a_2', a_3', and the inradius is $R+r'$ or $2R'+r'$, as desired.

ON CERTAIN PROJECTIVE TROCHOIDS

By R. M. WINGER, University of Washington

1. *Introduction.* Elsewhere[3] I have shown that the maximum axial symmetry which an algebraic curve of order m can admit is m-fold, namely symmetry with respect to m equispaced lines about a point. The equations of rational curves with this maximum symmetry have been obtained.[4] From the form of the equations it appears that for every value of m there is a finite number of such curves which are projectively distinct. We naturally ask concerning rational curves having multiple symmetry which falls short of the maximum. Some information is already available, for symmetry with respect to k lines ($k>1$) implies the existence of a dihedral collineation group G_{2k} of order $2k$ under which the curve is invariant.[4] And when the curve is rational, it has been proved that k is odd when m is odd.[1] Hence

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[1] Ibid, Page 189.

[2] This theorem (with references) is given by Johnson, page 192, but the proof is omitted as "long and dull."

[3] American Mathematical Monthly, vol. 37, 1930, p. 4. A typographical error occurs in the theorem as stated, where the number of axes is given as $2n$, but the theorem is correctly invoked on p. 5.

[4] Winger, l. c., also, American Journal of Mathematics, vol. 36, 1914, pp. 66, 57.

A rational curve of order m which has $m-1$ axes of symmetry is necessarily of even order.

We shall prove that such curves are trochoids,[2] and that there is a one-parameter family of them for every (even) value of $m(>2)$. There is also a family of rational trochoids for each even value of $m(>2)$ having $m-2$ axes of symmetry. These two families form the subject matter of this paper.

Trochoids, which are also epicycles, have occupied the attention of a host of mathematicians[3] since their invention by the ancients to account for the apparent motion of the planets, and elaborate collections of figures have been published.[4] But most of the results are metric. We shall give the right of way to the projective view, indicating however the metrical connections. We use *projective trochoid* to signify a curve that can be projected into a trochoid. We restrict the discussion to those curves which exhibit in their metrical setting the two highest orders of symmetry but results are stated for general n. Specifically, we deal with projective trochoids which are (a) rational, (b) of even[5] order $2n$, and (c) invariant under dihedral collineation groups of orders $2(2n-1)$ and $2(2n-2)$. The paper illustrates admirably the effectiveness of the group and projective attack in studying the singularities of invariant curves and particularly in resolving the higher singularities that occur in the infinite region. The two families of curves also exemplify the difference between curves having an odd and even number of axes of symmetry. I shall assume such knowledge of dihedral groups, binary and ternary, as is contained in my book on Projective Geometry, §§159, 160 and adopt the same notation and nomenclature.

I. *The Projective Trochoid C_{2n} Invariant Under a Dihedral $G_{2(2n-1)}$.*

2. *The parametric equations of the curve.* Since the curve is rational, two collineation groups are involved. For while the points of the curve are being permuted according to the transformations of the ternary group, the parameters of the points are permuted by a binary group, simply isomorphic with the first.

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[1] Winger, *Self-Projective Rational Septimics*, American Journal, vol. 47, 1925, p. 208.

[2] We use trochoids in the generic sense as including epi- and hypo-trochoids and of course epi- and hypo-cycloids which are denoted by the general term cycloids.

[3] The reader will find systematic accounts in Proctor, *The Geometry of Cycloids*, where reference is made to DeMorgan's essay in the Penny Encyclopaedia, Wieleitner, *Spezielle Ebene Kurven*, Loria, *Spezielle Algebraische und Transzendente Ebene Kurven*, Band 2. Moritz, *Cyclic-Harmonic Curves*, University of Washington Publications in Mathematics, treats some special trochoids, giving numerous figures. Instructive papers have been written by Morley, *On the Epicycloid*, American Journal of Mathematics, vol. 13, 1891, also *On Adjustable Cycloidal and Trochoidal Curves*, Ibid. vol. 16, 1894 and by Wolstenholme, *On Epicycloids and Hypocycloids*, Proceedings, London Mathematical Society, vol. 4, also by Roberts, *On Epi- and Hypo-trochoids*, Ibid., vol. 4, p. 353. For a digest of the literature and an extensive bibliography, see Wölffing, Bibliotheca Mathematica, vol. 2 (1901).

[4] H. Perigal, *Contributions to Kinematics*, for example.

[5] It is well known that algebraic trochoids are both rational and of even order. See the article by Roberts, cited in footnote 3, where these facts are proved.

Likewise the line sections of the curve must be permuted by the associated groups—the points by the ternary, the parameters by the binary group. In particular the points cut from the curve by any fixed line must be permuted among themselves.

The general parametric equations of a rational $2n$-*ic* are

$$x_0 = f_0(t),\ x_1 = f_1(t),\ x_2 = f_2(t),$$

where f_i are polynomials in t of order $2n$. The problem is to restrict these equations so that the curve will admit the group. We take for triangle of reference the fixed triangle of the group and we select as generators of the binary group

$$S{:}t' = \epsilon t, \text{ and } T{:}t' = 1/t,$$

ϵ a primitive $(2n-1)$th root of unity. Now each side of the reference triangle is fixed under the invariant cyclic subgroup of index 2 of the ternary group, while one side (say x_2) is fixed and the other two are interchanged by the reflexions. Accordingly S must carry f_i into themselves, while T must interchange f_0 and f_1 and leave f_2 unaltered—except possibly for a constant factor. We are thus led to the canonical equations of our curve:

$$(1) \qquad x_0 = t^{2n} + at,\ x_1 = at^{2n-1} + 1,\ x_2 = t^n,\ a \neq 0,\ \infty,$$

of which the line equations are

$$(2) \qquad \begin{aligned} \xi_0 &= (n-1)at^{3n-2} - nt^{n-1} \\ \xi_1 &= -nt^{3n-1} + (n-1)at^n \\ \xi_2 &= at^{4n-2} + 2\left[n - a^2(n-1)\right]t^{2n-1} + a. \end{aligned}$$

We thus get a family of curves since the systems of equations (1) and (2) contain one arbitrary constant. We shall first treat the general curve of the family and then notice some particular cases.

3. *The General Curve of the Family.* From (1) it appears that x_2 is a multiple line, having n-point contact at each of the points whose parameters are $0, \infty$, absorbing thus $2n-4$ points of inflexion. Again from (2) we see that the class[1] of x_2 is $2n-2$ so that it absorbs $(n-1)(2n-3)$ double lines including flexes. Or,

x_2 is equivalent to $2n-4$ flex tangents and $2n^2-7n+7$ bitangents.

The Plücker numbers for the curve, deducting the flexes and bitangents absorbed by x_2 are

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[1] The class of a multiple line is the dual of the order of a multiple point. A general line through a multiple point of order k cuts the curve in k coincident points, and a multiple line of class k counts for k coincident tangents from a general point of the line.

$\nu = 2(2n-1)$, the same as the order of the group
$d = (n-1)(2n-1)$
$\delta = 4(n-1)(2n-3) - (2n^2-7n+7) = (2n-1)(3n-5)$
$\rho = 6(n-1) - 2(n-2) = 2(2n-1) = \nu$,

where ν denotes the class, d the number of double points, δ the number of bitangents and ρ the number of flexes.

The special sets of parameters conjugate under the binary group are easily accounted for: The special pair, $0, \infty$, are the contacts of the multiple line while the two special sets of $2n-1$ parameters, namely

$$t^{2n-1} + 1 = 0, \quad t^{2n-1} - 1 = 0 \tag{3}$$

comprise in the aggregate the pairs of double points of the $2n-1$ involutions, viz.

$$t^2 - \epsilon^{2i} = 0, \quad i = 0, 1, \cdots, 2n-2. \tag{4}$$

Thus one double point of each involution belongs in each special set (3). Moreover *the double points* (4) *lie in pairs on the axes of reflexion*, $x_0 - \epsilon^i x_1 = 0$ *and hence name contacts of tangents from the corresponding centers*.

Since an involution has but two double points, the parameters of the $2n-2$ remaining intersections of each axis must be interchanged by an involution while the points themselves are fixed under the associated reflexion, hence all such points are nodes. Thus

The double points of the curve are distributed equally on the axes of reflexion, $n-1$ *lying on each*. This accounts for all of the double points.

Since all the centers of reflexion lie on the multiple line, this line counts for $2n-2$ tangents from each center. Two others, as we have seen, have contacts on the corresponding axis at the double points of an involution. We are left then with $2n-2$ tangents from each center and these must unite to form double lines, since their contacts must be conjugate pairs of an involution. Or

From each center of reflexion run $n-1$ *double lines—accounting altogether for* $(n-1)(2n-1)$ *double lines*. The residual $2(2n-1)(n-2)$ double lines must be distributed in $n-2$ general conjugate sets.

The flex equation, exclusive of $0, \infty$, each of which is an $(n-2)$-fold root is[1]

$$an(n-1)t^{4n-2} + 2[a^2(n-1)^3 - n^3]t^{2n-1} + an(n-1) = 0. \tag{5}$$

Since all special sets of parameters have been accounted for, *the flexes must comprise a general set of conjugate points*.

The discriminant of equation (5) is a power of

$$[a^2(n-1)^3 - n^3]^2 - a^2n^2(n-1)^2. \tag{6}$$

[1] This is a second proof that the points $0, \infty$ absorb $n-2$ flexes each. They are however simple flexes when $n=3$.

If two flexes (5) coincide, (6) must vanish, i.e.

$$a^2(n-1)^3 - n^3 \pm an(n-1) = 0,$$

whence

$$a = \pm n/(n-1),\ \pm n^2/(n-1)^2. \tag{7}$$

Because of the symmetry however, if two flexes coincide, the whole set will coincide in pairs. The two curves corresponding to either pair of values of a in (7) are projectively equivalent: when $a = \mp n/(n-1)$, the curve has a cusp and when $a = \pm n^2/(n-1)^2$, the curve has an undulation at each of the points given by $t^{2n-1} \pm 1 = 0$.

The pencil of invariant conics

$$x_0 x_1 = \lambda x_2^2 \tag{8}$$

have contacts with each other at 0, ∞—with the multiple line as chord of contact. In addition each contains a set of points conjugate alike under the binary and the ternary group.[1] Noteworthy among these conics are the following:

(a) $n-1$ each cutting out $2n-1$ double points
(b) $n-1$, each touching the $4n-2$ tangents of the sets of double points in (a)
(c) two each with $2n-1$ contacts
(d) $n-1$ each touching $2n-1$ double lines
(e) $n-1$ each cutting out $4n-2$ contacts of the sets of double lines in (d)
(f) One cutting out the $4n-2$ flexes
(g) One touching all the flex tangents.

4. *Special Curves.*

A Curve with $2n-1$ *Cusps.* If $a = n/(n-1)$, the line equations of the curve become, on factoring out the cusp form:[2]

$$\xi_0 = t^{n-1},\ \xi_1 = -t^n,\ \xi_2 = (t^{2n-1} - 1)/(n-1). \tag{9}$$

The class is reduced to $2n-1$ and all of the flexes and double lines are absorbed by the cusps. We are left however with $(n-2)(2n-1)$ double points, which lie in sets of $n-2$ on the axes of reflexion. One cusp falls on each axis, to which it is tangent, while the whole set of cusps lie on one of the invariant conics (8). The curve admits a group of the maximum order for its class, being the dual of a curve of order $2n-1$ admitting a group of double the order.[3]

A Curve with $2n-1$ *Undulations*, obtained when $a = n^2/(n-1)^2$. All of the flexes are absorbed but the class and number of the double points is unchanged.

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[1] The sets may be special for both groups as in (c), or the points may be a special set of the ternary group and their parameters form a general set of the binary group as in (a).

[2] When $a = -n/(n-1)$, we get a projectively equivalent curve with cusps at $t^{2n-1} + 1 = 0$.

[3] The equations of a rational curve of order n invariant under a dihedral group of order $2n$ are

$$x_0 = t^r,\ x_1 = t^s,\ x_2 = t^n + 1,\ r + s = n,\ 1 \leqq r \neq s$$

See my first paper in the American Journal, cited on p. 578.

Each undulation tangent counts as a double line so that the number of proper bitangents is reduced to $3(n-2)(2n-1)$. Each axis of reflexion contains one undulation, whose tangent passes through the corresponding center. The undulation tangent counts for three simple tangents from a center of reflexion. There is one simple tangent from the center with its contact on the axis. The other tangents from the center, exclusive of the multiple line, unite to form bitangents, hence

From each center of reflexion run one simple tangent and one undulation tangent, each with contact on the axis—and $n-2$ bitangents.

There is one conic of the invariant pencil touching the curve at each undulation, so that the undulation tangents account for all of the common lines of the two curves.

$a=1$. The curve now has a $(2n-1)$-fold point at ξ_2 which absorbs all of the double points. Each tangent of the multiple point passes through one center of reflexion and has its contact on the corresponding axis. The remaining point of intersection of each axis is a contact of tangent from a center, the aggregate of these points comprising one of the special sets of the binary group. The other special set of $2n-1$ points are the parameters of the multiple point. The flexes and bitangents are disposed as in the general case.

5. *Metrical Specialization.*

The curve (1) may be given a metrical setting. First we interpret the x's as homogeneous Cartesian coordinates X', Y', Z', writing $x_0=X'$, $x_1=Y'$, $x_2=Z'$, where $Z'=0$ is the equation of the line at infinity.[1] Then we change to ordinary rectangular coordinates X, Y by the transformations

$$X'/Z' = X,\ Y'/Z' = Y.$$

Finally we transform to circular or absolute coordinates by the substitutions

$$x = X + iY,\ \bar{x} = X - iY,\ i = \sqrt{-1},$$

and interpret the parameter as a complex number of absolute value 1, thus letting t run around the unit circle in the complex plane. None of these operations will affect the projective properties of the curve whose equations now become

$$x = (t^{2n} + at)/t^n,\ \bar{x} = (at^{2n-1} + 1)/t^n,\ t = e^{i\theta}. \tag{10}$$

In this form the curve exhibits symmetry with respect to $2n-1$ equispaced lines, namely the axes of reflexion $x^{2n-1}-\bar{x}^{2n-1}=0$. Further it appears that the curve is a trochoid.[2] Since this curve is the only rational curve of even order admitting a dihedral group of the order here in question, we have proved:

If a rational curve of order $2n$ is invariant under a dihedral group of order

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[1] Cf. *Projective Geometry*, §28.

[2] Cf. Harkness and Morley, *Theory of Functions*, §25. Also, Morley, American Journal, l. c.

$2(2n-1)$, *it is projectively equivalent to a trochoid having symmetry with respect to* $2n-1$ *lines.*

The trochoidal equations will perhaps be more familiar in rectangular coordinates. Instead of (10), let us write

$$x = bt^n + at^{1-n}, \tag{11}$$

a change which affects only the size. Then since $t=\cos\theta+i\sin\theta$, we have by De Moivre's theorem

$$x \equiv X + iY = b(\cos n\theta + i\sin n\theta) + a[\cos(1-n)\theta + i\sin(1-n)\theta].$$

Equating real and imaginary parts, we get the parametric equations in rectangular form

$$\begin{aligned} X &= b\cos n\theta + a\cos(1-n)\theta \\ Y &= b\sin n\theta + a\sin(1-n)\theta. \end{aligned} \tag{12}$$

This is the form given by Williamson[1] for an epicyclic which he identifies with a trochoid.

In particular, let $a=b$ and to avoid fractions put $\theta=2\omega$. Then by a trigonometric identity (12) reduces to

$$\begin{aligned} X &= 2b\cos(2n-1)\omega\cos\omega \\ Y &= 2b\cos(2n-1)\omega\sin\omega \end{aligned} \tag{13}$$

which leads at once to the polar equation

$$\rho = 2b\cos(2n-1)\omega,$$

a rose curve of $2n-1$ leaves.

In this metrical specialization, the multiple line is the line at infinity and the contacts fall at the circular points. In addition to their symmetry the trochoidal curves will have all of the projective properties relating to flexes, double points and double lines noted in §§3, 4. The pencil of invariant conics however are now concentric circles. In the family of trochoids (11) there is obviously one cycloidal curve: this is the curve with $2n-1$ cusps, for the condition, $h=\pm r$, that the tracing point be on the rolling circle is the same as the cusp condition, $a/b=\pm n/(n-1)$.

II. *The Projective Trochoid* C_{2n} *Invariant under a Dihedral* $G_{2(2n-2)}$.

6. *The General Curve of the Family.* As in §2 the canonical equations of a rational projective trochoid of order $2n$ invariant under a dihedral group of order $2(2n-2)$ are found to be

[1] *Differential Calculus*, Seventh Edition, p. 366. From (12) may be obtained the equations of a trochoid in terms of the radii of the two circles by writing $n\theta=\phi$, $a=\mp h$, $b=R\pm r$, $(1-n)/n=\pm(R\pm r)/r$, where R is the radius of the fixed, r that of the rolling circle and h is the distance of the tracing point from the center of the rolling circle.

$$x_0 = t^{2n} + at^2,\ x_1 = at^{2n-2} + 1,\ x_2 = t^n,\ a \neq 0, \infty, n \text{ odd}^1. \tag{14}$$

Again we have a family of such curves. In this section we shall discuss the general curve of the family. The line equations are

$$\begin{aligned} \xi_0 &= (n-2)at^{3n-4} - nt^{n-2}, \\ \xi_1 &= -nt^{3n-2} + (n-2)at^n, \\ \xi_2 &= 2\{at^{4n-4} + [n - a^2(n-2)]t^{2n-2} + a\}, \end{aligned} \tag{15}$$

$0, \infty$ factoring out, Thus the class of the curve is $4n-4$.

The special pair of parameters, $0, \infty$, conjugate under the binary group name cusps of which x_2, the fixed line of the ternary group, is the common tangent, having n-point contact at each. Each counts once as a cusp in reducing the class, but when $n>3$ each absorbs extra nodes, flexes and bitangents, the enumeration of which will be a special task of the immediate sequel.

Each axis of reflexion (aside from x_2) cuts out the double points of an involution, which are the parameters of contacts of simple tangents from the center of the associated reflexion. The double points of these involutions together make up the two special sets of $2n-2$ parameters of the binary group. The residual intersections of an axis must unite in pairs, forming double points. Or,

Each axis of reflexion (except x_2) carries $n-1$ nodes. There remain $(n-1)(2n-1)-2(n-1)^2=n-1$ double points. The parameters of these cannot form a special set under the binary group, for the three special sets have been accounted for. We conclude that the missing double points must fall at the two cusps—and they must be equally apportioned since the cusps are conjugate. Thus each cusp absorbs $(n-1)/2$ nodes, i.e. the *excess*[2] of nodes over the normal for each cusp is $(n-3)/2$.

The two cusps also exact a heavy toll of flexes and bitangents, which may be assigned to four causes: (a) each counts once as a cusp, (b) the tangent at each is a multiple line with n-point contact, (c) each contains latent double points and therefore latent double lines as well, (d) the two have a common tangent. We shall ascertain the total reduction in double lines as well as the number that is to be ascribed to each cause. First, two ordinary cusps on a rational curve of order $2n$ absorb together 4 flexes and $8n-13$ bitangents, as indicated by Plücker's equations. But from the flex equation

$$t^{n-1}\{n(n-2)at^{4n-4} + [(n-2)^3a^2 - n^3]t^{2n-2} + n(n-2)a\} = 0 \tag{16}$$

we infer that each cusp of our curve absorbs $n-1$ flexes, i.e. the excess of flexes in each cusp is $n-3$. Now each cusp is of class $n-2$ and as such would be equivalent to $(n-2)(n-3)/2$ double lines. But since the contacts of the tangent all coincide, $n-3$ of these double lines are flexes. To this number must be added

[1] If n were even the order of the curve would be reduced to n.

[2] Also called the number of *latent* double points.

$(n-3)/2$ double lines—all bitangents—by way of excess, since the number of latent double lines must match the number of latent double points.[1] Thus,

Each special cusp is equivalent to one ordinary cusp, $(n-3)/2$ nodes, $n-3$ flexes and $(n-3)^2/2$ bitangents—and these numbers are in addition to the usual quota of flexes and bitangents.

Indeed we must expect a further reduction in the number of bitangents since x_2 is a common tangent to the cusps. Now x_2 as a $(2n-4)$-fold line of a curve of order $2n$ would be equivalent to $(n-2)(2n-5)$ double lines. Adding to this the excess of $n-3$ for both cusps, the total is $2n^2-8n+7$. But in the total are $2(n-3)$ flexes, hence

x_2 is equivalent to $2n^2-10n+13$ bitangents and $2(n-3)$ flexes.

Removing the $(n-3)^2$ which the two cusps would absorb if they had separate tangents, we find the number of new bitangents consumed by the double cusp tangent to be $(n-2)^2$. The mortality of double lines occasioned by the two cusps and their common tangent due to each cause we may now tabulate:

Cause	*Flexes Absorbed*	*Bitangents Absorbed*
(a)	4	$8n-13$
(b)	$2(n-3)$	$(n-3)(n-4)$
(c)	0	$n-3$
(d)	0	$(n-2)^2$
Total	$2(n-1)$	$2n(n-1)$.

Deducting the number of singularities absorbed, we obtain the Plücker numbers for the curve:

$\nu=4(n-1)$, the same as the order of the group

$d=2(n-1)^2$

$\delta=8n^2-20n+12-2n(n-1)=6(n-1)(n-2)$

$\rho=4(n-1)=\nu$.

Since all special sets of the binary group have been accounted for, *the remaining flexes belong to a general conjugate set.*

We shall consider next the distribution of the bitangents. Since the invariant cyclic subgroup of index two is of even order, it contains one reflexion (corresponding to the involution $t'=-t$) whose center is ξ_2 and whose axis is x_2. Now all of the intersections of the axis fall at the two cusps, hence all of the tangents from the center must be bitangents, for the contacts must be interchanged by the involution. Or,

$2(n-1)$ *bitangents meet at ξ_2 and constitute a special set of lines under the ternary group while their parameters form a general set under the binary group.*

We have seen that from each center of reflexion on x_2 run two simple tangents with contacts on the corresponding axis, while the line x_2 itself counts for $2n-4$ others. The remaining ones will in general go to the formation of bitan-

[1] Cf. Scott, American Journal of Mathematics, vol. 15, 1893, p. 230 ff. Also, Winger, *Ibid.* vol. 47, 1925, p. 210.

gents. Hence, *from each of the* $2n-2$ *centers of reflexion on* x_2 *run two simple tangents, with contacts on the corresponding axis, and* $n-1$ *bitangents.*

This accounts for $2(n-1)^2$ additional double lines. There remain $6(n-1)(n-2)-2(n-1)-2(n-1)^2=4(n-1)(n-3)$ bitangents which must be distributed in $n-3$ general conjugate sets.

The pencil of invariant conics contains several members analogous to those enumerated in Part I.

7. *Special Cases.*

The discriminant of the flex equation (16), exclusive of the parameters of the cusps, is a power of the left side of

$$[(n-2)^3a^3-n^3]^2-4a^2n^2(n-2)^2=0. \tag{17}$$

Factoring this we have

$$(n-2)^3a^3-n^3=\pm 2an(n-2),$$

whence $a=\pm n/(n-2)$, $\pm n^2/(n-2)^2$. If $a=\mp n/(n-2)$, the curve has cusps and if $a=\pm n^2/(n-2)^2$ the curve has undulations at $t^{2n-2}\pm 1=0$. The two curves of either pair are projectively equivalent, hence it will suffice to discuss one of each.

A curve with $2n-2$ *additional cusps.* $a=n/(n-2)$. The line equations (15), on removing the cusp factor, reduce to

$$\xi_0=t^{n-2},\ \xi_1=-t^n,\ \xi_2=2(t^{2n-2}-1)/(n-2). \tag{18}$$

This curve admits the maximum dihedral group for its class since it is the projective dual of a curve of order $2n-2$ admitting the maximum group, for that order.[1] It is not however the only curve of its class invariant under the same group when $n>4$.

The cusps absorb all of the flexes and all but $n-1$ of the bitangents. The $2n-2$ new cusps lie in pairs on $x_0-x_1=0$ and the axes conjugate with it.[2] The curve thus has $n-1$ double cusp tangents, which are in fact the bitangents which are left. These double cusp tangents comprise all of the tangents to the curve from the center ξ_2, while all of the tangents from the other centers of reflexion are concentrated in the multiple line x_2. The entire set of $2n$ cusps lie on a conic.

A curve with $2n-2$ *undulations.* $a=-n^2/(n-2)^2$. The undulations absorb all of the flexes and $2n-2$ bitangents. They lie in pairs on the set of $n-1$ axes conjugate with $x_0-x_1=0$. The tangents at the two undulations on each axis meet at the corresponding center and replace two of the double lines in the

[1] See footnote, § 4.

[2] In a dihedral D_{2n}, when $n=2m$, the $2m$ axes of reflexion $x_0^{2m}-x_1^{2m}=0$ divide into two conjugate sets $x_0^m+x_1^m=0$, $x_0^m-x_1^m=0$. So also do the centers which lie one on each axis. Further the centers belonging to one set of axes lie on the other set of axes when m is odd, but on their own set when m is even. The statement in my *Projective Geometry*, p. 339 is inaccurate.

general case. One of the invariant conics, namely $(n-2)^4x_0x_1-16(n-1)^2x_2^2=0$, touches the curve at each undulation so that the undulation tangents account for all of the common lines of curve and conic.

$a=\pm 1$. The curve now has a $(2n-2)$-fold point at ξ_2, which would commonly absorb $(n-1)(2n-3)$ double points and leave $n-1$. But in the multiple point are $n-1$ tacnodes which use up the remaining $n-1$ double points. The multiple point thus has only $n-1$ distinct tangents, which are the axes $x_0^{n-1}+x_1^{n-1}=0$ or $x_0^{n-1}-x_1^{n-1}=0$, according as $a=+1$ or -1. Each tacnodal tangent counts once as a double line and absorbs a second double line on account of excess. This leaves $2(n-1)(3n-7)$ double lines to be accounted for. All of the tangents from ξ_2 are included in the tacnodal tangents. From each center of one conjugate set of $n-1$ reflexions run one tacnodal tangent, whose parameters are the double points of an involution, and $n-1$ bitangents. From each of the centers in the other set run two simple tangents and $n-1$ bitangents besides the multiple line x_2. We have thus accounted for $2(n-1)^2$ additional bitangents. There remain $4(n-1)(n-3)$ bitangents, which belong to $n-3$ general conjugate sets.

8. *Metrical Specialization.*

As in Part I, we may write the equations of the curve (14) in circular coordinates

$$x = t^n + at^{2-n},\ \bar{x} = at^{n-2} + t^{-n}, \tag{19}$$

which represent a genuine trochoid with the special cusps falling at the circular points and with the line at infinity as the common cusp tangent. From the first equation (19), we obtain as before the parametric equations in rectangular coordinates

$$\begin{aligned} X &= \cos n\theta + a\cos(n-2)\theta \\ Y &= \sin n\theta - a\sin(n-2)\theta, \end{aligned} \tag{20}$$

n a positive odd integer.
Or, writing $n\theta=\phi$, (20) takes the form

$$\begin{aligned} X &= \cos\phi + a\cos[(n-2)\phi/n] \\ Y &= \sin\phi - a\sin[(n-2)\phi/n]. \end{aligned} \tag{21}$$

When $a=n/(n-2)$, (21) reduces to a cycloid with $2n-2$ cusps—in addition to the special singularities at I and J. The cusps lie on a circle with center at the center of the curve and of radius $(2n-2)/(n-2)$.

When $a=1$, (20) becomes by a trigonometric identity

$$\begin{aligned} X &= 2\cos(n-1)\theta\cos\theta, \\ Y &= 2\cos(n-1)\theta\sin\theta, \end{aligned} \tag{22}$$

whence we get at once the polar equation

$$\rho = 2\cos(n-1)\theta,$$

a rose curve of $2n-2$ leaves.

Summary. It should be remarked that there are other trochoids admitting groups of lower orders. Our discussion includes however as special cases rose curves and cycloidal curves having the two highest orders of symmetry. On the other hand some of the curves are invariant under higher groups. For example when $n=a=3$, equation (19) represent the hypocycloid of class 4, commonly called the astroid, and which admits the octahedral group G_{24}. One lesson may be drawn from the present treatment: The cuspidal curves (rational cycloids) should be considered as class curves for then their parametric equations assume the simplest possible form, as exemplified in (9) and (18).

QUESTIONS, DISCUSSIONS, AND NOTES

EDITED BY R. E. GILMAN, Brown University, Providence, Rhode Island

The department of Questions and Discussions in the Monthly is open to all forms of activity in collegiate mathematics, including the teaching of mathematics, except for specific problems, especially new problems, which are reserved for the department of Problems and Solutions.

A PROBLEM IN INTEGRATION

Professor H. S. Uhler of Yale University writes as follows:

"I am taking the liberty of asking you to propose the following problem in integration since I can not find anything satisfactory about it, and since I need the results in my work. I have run across the *indefinite* integrals whose integrands are $\cos\theta\cos(\cot\theta)d\theta$, $\sin\theta\cos(\cot\theta)d\theta$, $\cos\theta\sin(\cot\theta)d\theta$, and $\sin\theta\sin(\cot\theta)d\theta$. The limits are θ_0 and θ where neither angle needs to exceed π radians. What I desire to know is whether these integrals can be expressed in closed form in terms of any known functions. If elliptic functions, and the like, are not involved then I should like to have series expansions given together with their respective limits or regions of convergence."

Will some reader furnish the answer.

R.E.G.

A NOTE ON DETERMINANTS

By C. O. OAKLEY, Brown University

PROPOSITION. If in a determinant

$$\Delta = |a_{ij}|, \quad \sum_{i=1}^{n} a_{ij} = \sigma, \ (j = 1, 2, \cdots, n)$$

and

$$\sum_{j=1}^{n} a_{ij} = \tau, \ (i = 1, 2, \cdots, n),$$

then $\sigma=\tau$, since, upon summing all elements, $n\sigma=n\tau$.

DEFINITION I. A determinant is said to be a sigma-determinant if

$$\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = \sigma.^{1}$$

DEFINITION II. A determinant Δ is said to possess unit-property if the $2n$ determinants Δ', formed by replacing the elements of a column (row) of Δ by unity, are all equal.

THEOREM I. *A necessary and sufficient condition that a determinant* $\Delta(\neq 0)$ *possess unit-property is that it be a sigma-determinant.*

PROOF OF NECESSITY. We are given that Δ possesses unit-property.[2] That is

$$\Delta' = \begin{vmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1 & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & 1 & \cdots & a_{1n} \\ a_{21} & 1 & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{n1} & 1 & & a_{nn} \end{vmatrix} = \cdots = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdot & \cdot & & \cdot \\ \cdot & & & \\ \cdot & \cdot & & \cdot \\ 1 & 1 & \cdots & 1 \end{vmatrix}.$$

Making use of the first two determinants in this relation and interchanging the first two columns in the second determinant, we write

$$\begin{vmatrix} 1 & a_{12} & & \\ 1 & a_{22} & & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & & \cdot \\ 1 & \cdot & & a_{nn} \end{vmatrix} = - \begin{vmatrix} 1 & a_{11} & & \\ 1 & a_{21} & \cdot & \\ \cdot & & & \cdot \\ \cdot & & & \\ 1 & & & a_{nn} \end{vmatrix},$$

from which we get by transposition and addition,

$$\begin{vmatrix} 1 & a_{11} + a_{12} & & \\ 1 & a_{21} + a_{22} & \cdot & \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \\ 1 & a_{n1} + a_{n2} & & a_{nn} \end{vmatrix} = 0.$$

By adding the 3rd, 4th, $\cdots$, nth columns respectively to the second column this becomes[3]

$$(1) \qquad D_{12} = \begin{vmatrix} 1 & \sum a_{1j} & & \\ 1 & \sum a_{2j} & \cdot & \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \\ 1 & \sum a_{nj} & & a_{nn} \end{vmatrix} = 0.$$

[1] It is the determinant of a magic square in which the sums of the diagonals play no part. So far as the author is aware such determinants, the class of which overlaps those of symmetric and skew-symmetric determinants, have not been treated.

[2] It should be noted that we are really concerned here with the matrix of Δ rather than the value of Δ.

[3] The notation $D_{\mu\nu}$ shall indicate that from the set Δ', determinants have been chosen one of which involves units in the μth column and the other of which involves units in the νth column.

We now form the $n(n-1)/2$ relations (1) obtained by so treating every pair of the n determinants Δ' with columnar replacements. Each determinant $D_{\mu\nu}$ so formed will contain a column of units and a column of $\sum$'s. Laplace's development of these determinants by means of these two columns gives us the following system of $n(n-1)/2$ equations in $n(n-1)/2$ unknowns:

$$
\begin{aligned}
&\overset{12}{\alpha}_{12}A_{12} + \overset{12}{\alpha}_{13}A_{13} + \cdots + \overset{12}{\alpha}_{1n}A_{1n} + \overset{12}{\alpha}_{23}A_{23} + \cdots + \overset{12}{\alpha}_{n-1,n}\,A_{n-1,n} = 0,\\
&\overset{13}{\alpha}_{12}A_{12} + \overset{13}{\alpha}_{13}A_{13} + \cdots + \overset{13}{\alpha}_{1n}A_{1n} + \overset{13}{\alpha}_{23}A_{23} + \cdots + \overset{13}{\alpha}_{n-1,n}\,A_{n-1,n} = 0,\\
&\qquad\qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\overset{n-1,n}{\alpha}_{12}A_{12} + \overset{n-1,n}{\alpha}_{13}A_{13} + \cdots + \overset{n-1,n}{\alpha}_{1n}A_{1n} + \overset{n-1,n}{\alpha}_{23}A_{23} + \cdots + \overset{n-1,n}{\alpha}_{n-1,n}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad A_{n-1,n} = 0;
\end{aligned}
\tag{2}
$$

where

$$
A_{kl} = \begin{vmatrix} 1 & \sum_j a_{kj} \\ 1 & \sum_j a_{lj} \end{vmatrix}
$$

and where $\alpha_{kl}^{\mu\nu}$ is the algebraic complement of A_{kl} in $D_{\mu\nu}$. Now the determinant H of the coefficients $\alpha_{kl}^{\mu\nu}$ in this system (2) is a power of Δ since it is a reciprocal determinant formed from Δ. Therefore H cannot vanish and hence

$$
A_{12} = A_{13} = \cdots A_{n-1,n} = 0,
$$

that is

$$
\sum a_{1j} = \sum a_{2j} = \cdots = \sum a_{nj} = \tau.
$$

This proves the theorem for rows; a similar treatment will prove it for columns. Thus Δ is a sigma-determinant.

PROOF OF SUFFICIENCY. This part of the theorem follows at once from the following considerations. If in Δ we add the 2nd, 3rd, $\cdots$, nth columns respectively to the elements of the first column, we may write

$$
\Delta = \begin{vmatrix} a_{11} & & \\ & \ddots & \\ & & a_{nn} \end{vmatrix} = \begin{vmatrix} \sum a_{1j} & & \\ \vdots & \ddots & \\ \sum a_{nj} & & a_{nn} \end{vmatrix} = \sigma\Delta'. \tag{3}
$$

Clearly we could thus factor out a σ from any column or row. This completes the proof of the theorem. Moreover the sufficiency persists even though $\Delta = 0$. For so long as $\sigma \neq 0$ it can be factored out as in (3) above; and for the case $\sigma = 0$ we can proceed as follows. Add ϵ to the elements of the main diagonal of Δ and write[1]

$$
\Delta_\epsilon = \begin{vmatrix} a_{11}+\epsilon & a_{12} & \cdots & \\ a_{21} & a_{22}+\epsilon & & \\ \vdots & & \ddots & \\ & & & a_{nn}+\epsilon \end{vmatrix} = \epsilon\Delta_\epsilon'.
$$

[1] This method was suggested to the author by Professor R. E. Gilman.

Therefore $\Delta_\epsilon' = \Delta_\epsilon/\epsilon$ and, since Δ_ϵ is a polynomial of the nth degree in ϵ, $\lim \Delta_\epsilon'$ exists and equals the $\lim \Delta_\epsilon/\epsilon$ and the theorem is proved. It is of interest to note the following corollaries.

COROLLARY I. If $\Delta(=0 \text{ or } \neq 0)$ is a sigma-determinant such that $\sum_i a_{ij} = \sigma$ and if Δ' is the value of one of the determinants formed from Δ by replacing the elements of some column (or row) by units, then $\Delta = \sigma\Delta'$.

COROLLARY II. A sigma-determinant vanishes if $\sigma = 0$.

It may happen, however, in case $\Delta = 0$, that Δ possesses unit-property but is not a sigma-determinant. Hence, in that event, necessity fails. But if $\Delta = 0$ and possesses unit-property there exists a set of multipliers $\rho_1, \rho_2, \cdots, \rho_n, v_1, v_2, \cdots, v_n$ such that if we multiply the first row of Δ by ρ_1, the second by $\rho_2 \cdots$, the nth column by v_n, then this new determinant $\rho\Delta$, say, is a sigma-determinant. We illustrate this fact by the following example. The determinant

$$\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix} = 0$$

possesses unit-property $(\Delta' = 0)$ but is not a sigma-determinant as can readily be seen. The multipliers $\rho_1 = \rho_2 = \rho_3 = v_1 = v_2 = 1$, $v_2 = 2$ give

$$2\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 2 & -2 & 2 \\ -1 & 4 & -1 \end{vmatrix} = 0$$

which is a sigma-determinant $(\sigma = 2)$.

It is not difficult to prove the following

THEOREM II. *The product of two sigma-determinants is a sigma-determinant.*

COROLLARY. The adjoint of a sigma-determinant is a sigma-determinant.

It is an obvious remark that a theory of what might be called semi-sigma-determinants might be developed from the point of view of rows (columns) alone; that is to say we might require that $\sum_{j=1}^{n} a_{ij} = \sigma$, $(i = 1, 2, \cdots, n)$ but make no restriction as to the sums $\sum_i a_{ij}$ by columns. Indeed this is essentially what has been done in the methods here used, the proposition stated at the beginning being used to tie rows and columns together; and it is this point of view that leads to some of the more interesting applications of the theory to systems of linear equations. For example consider the system

$$\sum_{j=1}^{n} a_{ij}x_j = b, \qquad i = 1, 2, \cdots, n; \tag{4}$$

with the condition that

$$\sum_{j=1}^{n} a_{ij} = \sigma, \qquad i = 1, 2, \cdots, n.$$

Nothing is said about $\sum_i a_{ij}$. Then we may state the following theorem about the solution of (4):

THEOREM III. *If* $\sum_{j=1}^{n} a_{ij} = \sigma \neq 0$, *the solution of* (4) $(\Delta \neq 0)$ *is* $x_1 = x_2 = \cdots = x_n = b/\sigma$.

This follows at once from the fact that $\Delta = \sigma b \Delta'$ and hence $x_j = b\Delta'/\Delta = b/\sigma$.

PROOF BY VECTOR METHODS THAT EVERY REAL SURFACE WITH TWO SETS OF RULINGS IS A QUADRIC SURFACE

By B. F. KIMBALL, Schenectady, N. Y.

Take the two sets of rulings as the parametric curves $u = \text{constant}$ and $v = \text{constant}$. Let $\boldsymbol{r}_0(u)$ and $\boldsymbol{r}_1(u)$ be the position vectors of the rulings $v = 0$ and $v = 1$, where u is the arc-length along the ruling $v = 0$. Then the equation of the ruled surface may be written:

$$\boldsymbol{R}(u, v) = (1 - v)\boldsymbol{r}_0(u) + v\boldsymbol{r}_1(u). \tag{1}$$

Let $\boldsymbol{k}_0$ and $\boldsymbol{k}_1$ be unit direction vectors of the lines $\boldsymbol{r}_0$ and $\boldsymbol{r}_1$ with positive sense in the direction of increasing u. Since u measures the arc-length of $\boldsymbol{r}_0$, we have:

$$\boldsymbol{r}_0' = \boldsymbol{k}_0.$$

We write

$$\boldsymbol{r}_1' = h(u)\boldsymbol{k}_1$$

where $h(u)$ is a scalar equal to the length of the vector $\boldsymbol{r}_1'$. Thus

$$\boldsymbol{R}_u = (1 - v)\boldsymbol{k}_0 + vh(u)\boldsymbol{k}_1.$$

Now since the $v = \text{constant}$ curves are rulings, $\boldsymbol{R}(u, v_0)$ represents a straight line for constant $v = v_0$. Hence

$$\boldsymbol{R}_u \times \boldsymbol{R}_{uu} \equiv 0.$$

We have $\boldsymbol{R}_{uu} = vh'(u)\boldsymbol{k}_1$. Thus

$$\boldsymbol{R}_u \times \boldsymbol{R}_{uu} = v(1 - v)h'(u)(\boldsymbol{k}_0 \times \boldsymbol{k}_1) \equiv 0$$

for all u and v. If $\boldsymbol{k}_0 \times \boldsymbol{k}_1 = 0$, it is easily seen that the surface is a plane. If $\boldsymbol{k}_0 \times \boldsymbol{k}_1 \neq 0$, we have $h'(u) \equiv 0$. Then $\boldsymbol{r}_1' = h\boldsymbol{k}_1$ where h is constant and we may express $\boldsymbol{r}_0$ and $\boldsymbol{r}_1$ by the relations

$$\boldsymbol{r}_0 = \boldsymbol{k}_0 u - \boldsymbol{a},$$

$$\boldsymbol{r}_1 = h\boldsymbol{k}_1 u + \boldsymbol{b},$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constant vectors. Accordingly, substituting for $\boldsymbol{r}_0$ and $\boldsymbol{r}_1$ in equation (1) we have:

$$\boldsymbol{R}(u, v) = uv(h\boldsymbol{k}_1 - \boldsymbol{k}_0) + u\boldsymbol{k}_0 + v(\boldsymbol{a} + \boldsymbol{b}) - \boldsymbol{a}.$$

Write this equation in the form

$$\boldsymbol{R} = \boldsymbol{A}uv + \boldsymbol{B}u + \boldsymbol{C}v + \boldsymbol{D}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are constant vectors with scalar components A_1, A_2, A_3; B_1, B_2, B_3 etc. and the scalar components of the position vector $\boldsymbol{R}$ are x, y and z, The above vector equation is equivalent to the three scalar equations

$$\begin{aligned} x &= A_1uv + B_1u + C_1v + D_1 \\ y &= A_2uv + B_2u + C_2v + D_2 \\ z &= A_3uv + B_3u + C_3v + D_3. \end{aligned} \tag{2}$$

Eliminating u and v from these equations, one obtains a quadratic equation in x, y and z. Thus the surface in question is a quadric surface.

A Geometric Interpretation of Landen's Transformation[1]

By C. E. Rhodes, University of Cincinnati

The problem of determining the volume between a cone and a cylinder affords an excellent illustration of the use of Elliptic Integrals. In addition, it leads readily to a simple geometric interpretation of Landen's Transformation. The problem is a natural one, and can be applied in finding the volume of grain or crushed stone dumped into a cylindrical bin.

The mathematical statement of the problem is as follows: Given a right circular cylinder and a right circular cone (one nappe) with their axes parallel, and a plane perpendicular to these axes passing through the vertex R of the cone; to find the volume V inside the cylinder between the plane and the cone. For convenience, let the radius of the cylinder be unity, and let k denote the distance OR between the axis of the cylinder and the axis of the cone; ϕ, the angle between the axis of the cone and any element; and y, the variable length PR. (See figures.) The required volume is swept out by the variable triangle PQR as it revolves about the axis of the cone, P following the intersection of the cylinder and plane, and Q following the intersection of the cylinder and cone. The differential of volume is then the area of the triangle PQR multiplied by the differential of the distance through which its center of gravity moves. Denoting the angle ORP by α, we have

$$dV = \frac{\operatorname{ctn}\phi}{3}\, y^3 d\alpha. \tag{1}$$

For the case where $k<1$, (Fig. 1) α is a convenient variable of integration, and, noting the symmetry, the required volume can be expressed as

$$V = \frac{2\operatorname{ctn}\phi}{3}\int_0^{\pi} [(1 - k^2\sin^2\alpha)^{1/2} + k\cos\alpha]^3 d\alpha. \tag{2}$$

---

[1] This paper was presented in abridged form before the Ohio Section of the Mathematical Association of America at Columbus April 7, 1932.

When this integrand function is expanded, certain terms can be dropped because their integrals vanish.

The standard Legendre Type Forms of Elliptic Integrals of the first and second kinds are

$$F(k) = \int_0^{\pi/2} \frac{dt}{(1 - k^2 \sin^2 t)^{1/2}}, \quad E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 t)^{1/2} dt.$$

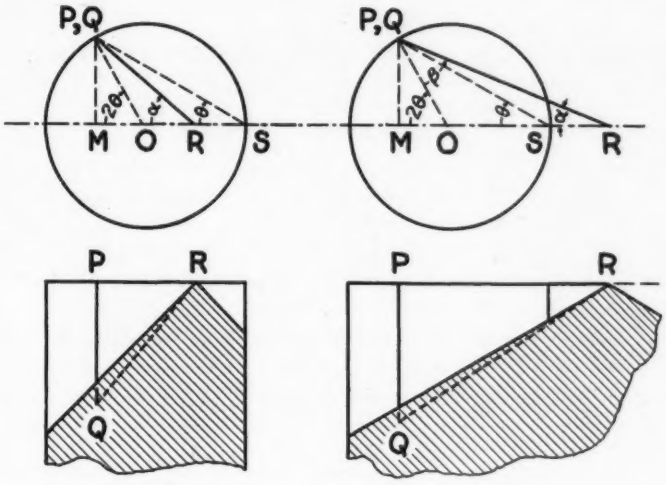

FIGURE 1

FIGURE 2

By means of the usual reductions,[1] the integral (2) can be expressed as

$$V = \frac{2 \operatorname{ctn} \phi}{9} \cdot 2[(7 + k^2)E(k) - 4(1 - k^2)F(k)] \tag{3}$$

where k is the modulus.

In considering the case shown in Fig. 2 where $k > 1$, α is not a convenient variable of integration, since it does not vary monotonically as the point P moves around the cylinder. On the other hand, the angles OPR and MOP both vary monotonically. Transforming the variable of integration in (1) from α to β, the volume integral can be written

$$V = \frac{2 \operatorname{ctn} \phi}{3} \int_0^{\pi} [(k^2 - \sin^2 \beta)^{1/2} + \cos \beta]^2 \frac{\cos \beta}{(k^2 - \sin^2 \beta)^{1/2}} d\beta \tag{4}$$

Reducing (4) to the Legendre Type Forms, it becomes

[1] Hancock, *Elliptic Integrals*, pages 61, 62.

$$(5) \qquad V = \frac{2 \operatorname{ctn} \phi}{9} \cdot \frac{2}{k}\left[k^2(7 + k^2)E\left(\frac{1}{k}\right) - (3 + k^2)(k^2 - 1)F\left(\frac{1}{k}\right)\right]$$

where the modulus is $1/k$.

In using the angle MOP as the variable of integration, it is found convenient in subsequent reductions to denote it by 2θ. Making the proper substitution in (1) the volume integral turns out to be

$$(6) \qquad V = \frac{2 \operatorname{ctn} \phi}{3} \int_0^{\pi/2} (1 + k^2 + 2k \cos 2\theta)^{1/2}(1 + k \cos 2\theta) 2d\theta.$$

Again applying the standard reductions, (6) becomes

$$(7) \qquad V = \frac{2 \operatorname{ctn} \phi}{9} (1 + k)[(7 + k^2)E(m) - (1 - k)^2F(m)]$$

in which the modulus is $m = 2k^{1/2}/(1+k)$. The form (7) is valid for both cases regardless of the size of k.

Landen's Transformation as used in Elliptic Integrals is defined by the equation $\tan \alpha = \sin 2\theta/(k+\cos 2\theta)$ and transforms any Elliptic Integral with modulus k or $1/k$ into one with modulus $2k^{1/2}/(1+k)$. From Fig. 1 it is evident that

$$\tan \alpha = \frac{MP}{MO + OR} = \frac{\sin 2\theta}{k + \cos 2\theta}$$

so that the transformation of the variable of integration from α to θ is actually Landen's Transformation. It can be shown that in Fig. 2, the change from β to θ is another example of it. The plane views of the problem furnish, then, a geometric picture of Landen's Transformation. To make the picture complete, the angle θ itself can be readily constructed as the angle MSP.

This particular geometric interpretation of Landen's Transformation is given by Dr. Hancock in his monograph[1] on Elliptic Integrals. There he obtains it as a special case of a much more complicated geometric transformation due to Jacoby. Here it is obtained as a natural result of finding the volume between a cylinder and a cone, using different variables of integration.

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[1] Loc. cit. page 74.

RECENT PUBLICATIONS

EDITED BY ROGER A. JOHNSON, Brooklyn College of the City of New York

All books for review should be sent directly to the editor of this department, at Brookly College, 66 Court Street, Brooklyn, N. Y., and not to any of the other editors or officers of the Association.

REVIEWS

La Géométrie. By Lucien Godeaux. Paris, Hermann et Cie, 1931. 181 pages. 15 francs (unbound).

In the first three chapters, about a half of the book, the author summarizes the basic ideas on geometry that have become current since the beginning of the present century: the relation of geometry to the notion of a group of transformations, the various interpretations of the non-Euclidean geometries and their interrelation, etc. Both the analytic and the synthetic approaches are used. The presentation is clear, orderly, and comprehensive, considering the space.

The last two chapters are devoted to algebraic geometry and the geometry on an algebraic variety. Little has been done to make the fundamental ideas of these subjects accessible to wider mathematical circles, and these contributions are quite welcome. The author thought it preferable to omit all bibliographical references from the text, a procedure which has its disadvantages. He compensates for the omission by a bibliography at the end of the book, in which he includes several American titles. The book may be read with profit by any student who is familiar with projective geometry.

NATHAN ALTSHILLER-COURT

Géomètres Français sous la Révolution. By Niels Nielsen. Copenhagen, Levin and Munksgaard, 1929. viii+250 pages. $2.50 (unbound).

It would seem a priori that the times of political changes and social upheavals are not auspicious for the cultivation of so abstract and contemplative a science as mathematics. But the period of the French Revolution is a striking example to the contrary. Prof. J. L. Coolidge recently[1] called the second quarter of the nineteenth century the "heroic age" in geometry. The period of the French Revolution may with justice be called the "romantic age" of mathematics. It is nothing short of romance to think that men like Carnot and Monge, to mention only two, who were in the very midst of the political and military turmoil, could turn their minds towards mathematical creation.

The late Professor Nielsen deserved well of all those interested in the history of mathematics for having given us along with the biographies of Fourier, Lagrange, Laplace, Carnot, Legendre, Monge, the biographical sketches of their lesser contemporaries. Each biography contains also a résumé of the most important contributions of the mathematician considered. That this was a labor of love is attested by the fact that of the seventy odd mathematicians whose

[1] Bulletin of the Am. Math. Society, 1929, pp. 1–19.

biographies are given, the names of more than half are not mentioned in Cajori's "History of Mathematics." The book was awarded a prize by the Paris Académie des Sciences.

NATHAN ALTSHILLER-COURT

Die Liesche Theorie der Partiellen Differentialgleichungen Erster Ordnung. By Friedrich Engel and Karl Faber. Leipzig, B. G. Teubner, 1932. xi+367 pages. RM 28.

The purpose of this text is to set forth with rigor and elegance the theory of linear partial differential equations of the first order from Lie's original viewpoint. The work of Professor Engel in connection with the editing of Lie's works has fitted him conspicuously for this task and the resulting text is a thoroughly commendable exposition of the Lie theory.

This text is elementary in the sense that only a restricted knowledge of mathematics is presupposed. The authors state these presuppositions as follows: some familiarity with ordinary differential equations, an adequate facility in handling determinants, and the ability to extend to n dimensions the analytic geometry of ordinary space. There is however assumed, on the part of the reader, a rather high degree of mathematical maturity and, to this extent, the text is by no means an elementary one.

The painstaking care of the authors in the preparation of the subject matter is in evidence on every page. The presentation is clear and concise and the arrangement thoroughly well thought out. Although the style is a trifle monotonous and the major results are not made to stand out as conspicuously as one would wish, the book is ably written and offers an excellent approach to this important subject.

C. H. SISAM

Vector Analysis, with an Introduction to Tensor Analysis. By A. P. Wills. New York, Prentice-Hall, Inc., 1931. xxxii+286 pages. $5.00.

This book falls naturally into two main sections: Chapters I to VI on vector analysis (including dyadics), Chapters VII to X on tensor analysis.

The first section follows, in general, the methods and notation of Gibbs, the principal exception arising in the definitions of divergence and curl of a vector function at a point as the limits of integrals over surfaces about the point, divided by the volume enclosed. The gradient of a scalar function is defined in the usual manner and then also by means of a limiting surface integral; the definitions are then shown to be consistent. While these integral definitions lend themselves to simple deductions of the standard integral transformations, they have the disadvantage of requiring a proof that the definitions themselves are unambiguous—a proof not easy to present in a brief and rigorous form. The reviewer personally prefers to define the divergence and curl of a vector point function $\boldsymbol{v}$ as the first scalar and vector invariant of the derivative dyadic $\nabla\boldsymbol{v}$.

Numerous applications of vector analysis are given in this portion of the book. These include the geometry of curves and surfaces in Euclidean 3-space, kinematics, potential theory and electrodynamics.

The second section on tensor analysis will be of particular interest to students of modern physics. Beginning with coordinate systems and transformations in 3-space, the results are then generalized to a Riemannian n-space. Then follows a discussion of geodesics in R_n, the parallelism of Levi-Civita, absolute differentiation and Riemannian curvature. The last chapter is devoted to the theory of tensors regarded as multilinear forms in n base-vectors, which are invariant under coordinate transformations. This point of view, initiated by Hessenberg in his Annalen article of volume 78, contrasts with the usual treatment of tensors in which their *components* alone appear—i.e. the scalar coefficients in the above forms. This treatment is especially happy in its direct approach to covariant differentiation. The book closes with a discussion of the Riemann-Christoffel and the Ricci-Einstein tensors.

A preliminary chapter of some twenty pages gives an excellent historical introduction to vector and tensor analysis. The book is well printed, and the few misprints that occur will cause the reader no particular difficulty. The last paragraph on page 229, however, should be corrected.

LOUIS BRAND

An Introduction to the Theory of Canonical Matrices. By H. W. Turnbull and A. C. Aitken. Glasgow, Blackie & Son, 1932. xiii+192 pages. 17sh. 6 d.

As stated in the preface, this book, though intended to serve as a sequel to Turnbull's *Theory of Determinants, Matrices, and Invariants*, published in 1928, has been made self-contained, presupposing merely some knowledge of the elementary theory of determinants. The scope of the book is indicated by the following list of chapter headings: I, Definitions and Fundamental Properties of Matrices; II, Elementary Transformations. Bilinear and Quadratic Forms; III, Canonical Reduction of Equivalent Matrices; IV, Subgroups of the Group of Equivalent Transformations; V, A Rational Canonical Form for the Collineatory Group; VI, The Classical Canonical Form for the Collineatory Group; VII, Congruent and Conjunctive Transformations; VIII, Canonical Reduction by Unitary and Orthogonal Transformations; IX, The Canonical Reduction of Pencils of Matrices; X, Applications of Canonical Forms to Solution of Linear Matrix Equations; XI, Practical Applications of Canonical Reduction.

The recent applications of matrix algebra to quantum mechanics by Dirac and others, makes the appearance of a book of this kind especially timely. The physicist who wishes to obtain without prolonged preliminary study a working knowledge of the theory of matrices of finite order will find the book adapted to his needs. The mathematician will observe that though the account of the theory presented here is neither so detailed nor complete as that given by Muth's *Elementarteiler* or the *Matrices and Determinoids* of Cullis (a fact in-

dicated by the authors), the work exhibits merits of its own. Among these should be noted the systematic use of the matrix notation, the interesting historical notes that conclude each chapter, and the numerous examples that "are intended to serve not so much as exercises, many of them being quite easy, but points of relaxation, and running commentary; they will, however, be found to contain many well-known and important theorems, which the notation establishes in the minimum of space."

While one may regret the paucity of geometrical applications given by the authors, the applications of the theory contained in the last two chapters of the book are well-chosen and interestingly presented.

L. M. BLUMENTHAL

Astronomy. By Forest Ray Moulton. New York, The Macmillan Company, 1931. xxiii+549 pages; tables xxii pages. $3.75.

This book is not a revised edition of the author's "Introduction to Astronomy," but an entirely new publication.

The subject matter has been taken up in such a way as "to unfold the subject of astronomy by proceeding step by step from the familiar earth and evening constellations out to the galaxies and super galaxies of stars."

A few pages of preliminary considerations are followed by a chapter on the constellations including stellar magnitudes and celestial coordinates. The succeeding chapters are on telescopes, the Earth, and the Moon, time, eclipses and the law of gravitation. What may be called the first part of the book is then concluded by chapters on the solar system, the planets, and meteors and comets. The last 200 pages of the text are devoted to the Sun, the evolution of the solar system, stars, nebulae, and the sidereal structure. The fact that only about one fourth of the book is on the stars is compensated for by the author's scholarly presentation of the subject matter which enabled him to include a wealth of information in a very compact form.

The questions and reference books given at the end of each chapter add greatly to the value of the book.

It is unfortunate that on the star maps the boundaries of the constellations are given in heavy solid lines which together with the rather bold designation of the stars makes it difficult to visualize the configurations. On Map I, "Path of Pole of Ecliptic" should read "Path of Pole of Equator."

The numerous diagrams and reproductions are clear and sharp.

The press work and binding are excellent, and notwithstanding the many pages and good paper the weight of the book is not excessive.

M. F. WEINRICH

The Size of the Universe. By Ludwik Silberstein. Oxford University Press, 1930. viii+216 papers. $3.50.

Dr. Silberstein has in this volume collected his views and investigations con-

cerning relativistic cosmology into a systematic treatment, which may perhaps be fairly said to sum up his stand on the subject up to the time of publication—late in 1929 or early in 1930. In this book Dr. Silberstein has ignored practically all other work done in the field, with the exception of the original papers of Einstein and de Sitter, and has conducted a most bitter polemic against certain of those—notably Hubble and Weyl—whose conclusions are incompatible with his own. Although it may not seem entirely fair to criticize the author's views of two or three years ago at this comparatively late date, the reviewer would justify his criticisms (but not his procrastination!) on the ground that they refer either to matters intrinsic to the book or to work which was in the literature at the time of publication—although he does not of course deny himself the opportunity of pointing out the more recent advances which have been made on the basis of this previous work.

In Part I, following an interesting diversion concerning amorphous and projective spaces, the author discusses in some detail the metrization of space and gives a brief account of the general theory of curvature of higher dimensional manifolds. Part II begins with the assumption that the universe of space and time can in the large be considered as *homogeneous*, but not necessarily *isotropic*—any event in this idealized space-time bears the same relation to the metrical structure as any other, but not all space-time directions through them are equivalent in the same sense. The author then derives as the cases in which he is interested the universes of Einstein and de Sitter (the latter of which is completely isotropic as well as homogeneous). His assumption of complete homogeneity has thus closed to him the more general non-stationary possibilities, first realized by Friedmann in 1922, of universes which are *spatially* isotropic and homogeneous, upon which the recent developments of Lemaître, de Sitter, Tolman, Eddington, Einstein, the reviewer and others are based (for an account of which the interested reader is referred to the reviewer's report *Relativistic Cosmology* to appear in the January, 1933, number of *Reviews of Modern Physics*).

The Einstein universe he emphatically discards because of the difficulty of introducing into it a mass without causing objectionable disturbances at its polar plane. But before abandoning it Dr. Silberstein indulges in a most inexcusably bitter polemic (Part III) directed against Hubble's interesting and important paper (*Astrophysical Journal*, vol. 64 (1926), pp. 321–369) on extragalactic nebulae, presumably because Dr. Hubble has the temerity to end his 48-page paper with a half page computation expressing the value of the density of matter in terms of the radius of the Einstein universe (with the aid of formulae from Haas's *Introduction to Theoretical Physics*, to which Dr. Silberstein refers (p. 98) as "a semi-popular book"!).

Part IV returns to a more complete discussion of the de Sitter universe, including an account of the motion of particles and light. For this purpose the author employs the original de Sitter coordinates, the inadequacy of which was recognized by Weyl in 1923, in place of the more appropriate coordinates in-

troduced by Lemaître in 1925. The last Part applies these results to a determination of the Doppler effect in light from distant objects; the general formula thus obtained depends on the perihelion distance and speed at perihelion of the star or nebula, as well as on its present distance and the "radius" $\mathfrak{R}$ of the de Sitter universe, but as these two quantities are not obtainable from the data he eliminates them with the aid of statistical assumptions to obtain an expression for the radius in terms of the mean square distance and Doppler effect of a group of objects. This statistical formula he applies to several groups of objects (stars, globular clusters and the Magellanic Clouds) in or closely associated with our galactic system and finds for the curvature radius values ranging from 1.46×10^6 to 3.09×10^7 parsecs, and to a group of 38 spiral nebulae with the result $\mathfrak{R}=7.32\times10^8$ parsecs. Postponing for a moment discussion of these results, we continue with a summary of topics included in the remainder of Part V and in the remaining Notes (Nos. 4, 4a and 8 containing estimates of $\mathfrak{R}$ included in the above limits). The orbit of particles in the field of a spherically symmetric mass introduced into the de Sitter universe are computed and the results interpreted with the aid of the intriguing concept of a "cosmic day." The remaining notes contain, in addition to a more detailed analysis of certain points in the body of the book, a most biting rejoinder to certain criticisms by Weyl of portions of the author's work, and an account of the author's questionable attempt (*Philosophical Magazine*, vol. 9 (1930), pp. 50–57) to derive the line element describing a homogeneous universe filled with isotropic radiation—in which the author apparently considers the cosmological constant λ as variable, contrary to the general theory of relativity, as he would otherwise obtain the rejected Einstein universe!

Since the most important fact to be explained here concerns the observed "Doppler shifts" in light from distant objects, a few words concerning the history and present status of this problem may not be out of place. The original development by de Sitter of the universe which bears his name included the prediction of a residual red shift which, interpreted as a velocity, would lead to the conclusion that for a sufficiently restricted range $v\sim r^2$. Weyl showed in 1923 (*Physik. Z.* 24, pp. 230–232) that the coordinates which were employed by de Sitter covered but a portion of the universe, and found it desirable to introduce some assumption concerning the natural state of motion of matter. The assumption which he chose was the seemingly natural one that on following back the world-lines of matter they would be found to converge in the remote past—that the actual universe contains only matter which is coherent in this sense. With the aid of this assumption Weyl deduced a velocity-distance relationship which is of the form $v=cr/\mathfrak{R}$ for objects at distances r which are small compared with the radius $\mathfrak{R}$ of the de Sitter universe. That this linear law is indeed satisfied by those extra-galactic objects, the spiral nebulae, to which we can most justifiably expect to apply it was fairly widely recognized at the time Dr. Silberstein's book was written, and has since been most conclusively verified by Hubble at Mt. Wilson. Assuming that his observed red shift is in

fact that predicted by Weyl, the radius of the de Sitter universe is found to be $\mathfrak{R}=1.7\times10^{27}$ cm. $=6\times10^{8}$ parsecs, which is some hundreds of times the value most favored by Dr. Silberstein. The discrepancy between these results is of course to be explained by the difference in theoretical background as well as by the difference in the objects to which the theory is applied. In the first place, Silberstein has applied his statistical formula (which is equally applicable to red *and to violet* shifts) to stars and groups of stars which form a part of the relatively closely knit galactic system and finds it verified—contrary to the findings of Lundmark (*Monthly Notices R. A. S.* vol. 84 (1924), pp. 747–770) and Strömberg (*Astrophysical Journal* vol. 61 (1925), pp. 353–362), whose work he entirely ignores in this account. And in the second place he has applied it to the extra-galactic nebulae, without being able to account for the overwhelming preponderance of red shifts, to obtain results at variance with those which he obtained from his previous investigations (although an attempt to reconcile them is made by questioning Hubble's estimates of distances)—in connection with which it is interesting to note that the red shifts subsequently dealt with by Hubble and Humason are found in nebulae whose distances are estimated to extend up to 4×10^{7} parsecs, many of which must therefore lie without the limits set by Silberstein to the entire de Sitter universe!

H. P. ROBERTSON

MATHEMATICS CLUBS

EDITED BY F. M. WEIDA, The George Washington University, Washington, D. C.

All reports of club activities, suggestions and topics for club programs, and material of interest to clubs should be sent to F. M. Weida, The George Washington University, Washington, D. C. All manuscript should be typewritten, with double spacing, and with margins at least one inch wide.

CLUB ACTIVITIES

1931–1932

THE PI MU EPSILON MATHEMATICAL FRATERNITY

Pi Mu Epsilon is an academic fraternity in institutions of university grade. Its primary aim is the advancement of mathematics and scholarship. It is a living, active, working fraternity of scholars in which the members are actively engaged in study and research and the exchange of ideas in the field of mathematical science.

Pi Mu Epsilon of the University of Illinois

The officers for 1931–1932 of our chapter were: Homer Dennis, Director; William Ted Martin, Vice Director; Lucretia Ann Mott, Recording Secretary; Harry E. Crull, Corresponding Secretary; Herbert M. Norris, Treasurer.

All officers were elected at the regular spring election, May 19, 1931. All the officers were elected by a vote of the members present at this meeting.

The University of Illinois chapter has forty-four active members and twelve new members who were initiated March 22, 1932, making a total of fifty-six members.

Those eligible to membership are undergraduate students who have an average of at least 4.5 in mathematics and have declared their intention to major or minor in mathematics and shall be

enrolled in or have taken at least one three hour course in mathematics requiring as a prerequisite Integral Calculus. Four graduate students and one member of the faculty may also be elected at the annual election.

The programs and meetings were as follows:

October 13, 1931: Business meeting.

October 27, 1931: "Real branches of algebraic curves" by Professor Brahana.

November 10, 1931: "The outline of the theory of functionals" by B. T. Darling, '34.

November 24, 1931: "Symmetric functions" by D. M. Brown. This talk was followed by a business meeting.

February 23, 1932: A short business meeting and the election of members.

March 8, 1932: Rushing party. At this meeting, Lucretia Ann Mott, '32, gave a talk on "Mathematical recreation."

March 11, 1932: Pledge luncheon.

March 22, 1932: Initiation banquet, Inman Hotel, Champaign, Illinois.

April 12, 1932: "Moebius leaves" by Dr. Wilson.

One of our biggest accomplishments of the year was the revision of our by-laws.

LUCRETIA ANN MOTT, *Recording Secretary*

Pi Mu Epsilon of Iowa State College

The officers for 1931–1932 were: C. R. Wells, Director; E. W. Carr, Vice Director; E. A. Howard, Treasurer; Marianne Preuss, Secretary; E. C. McCracken, Librarian; Professor D. L. Holl, Faculty Advisor; Dr. E. R. Smith, Head of the Department of Mathematics, Permanent Secretary.

We now have seventy-nine active members. We had two initiations the past year, one on November 18th, at which time we took in twelve new members; and the other on May 24th, when we initiated ten.

At the annual college Honor's Day in the fall, 1931, we awarded a check of $15 and a certificate to Guy E. Strong, a junior ceramics engineering student. We give this prize each year to the student having the highest scholastic average who has completed the course in calculus. He is also taken into Pi Mu Epsilon.

Pi Mu Epsilon sponsored the Mathematics Openhouse during the spring carnival, Veishea, May 5th, 6th and 7th. The main feature was the showing of a Relativity Movie.

The programs and meetings were as follows:

November 9, 1931: "Linear transformation" by E. A. Howard.

November 18, 1931: Initiation and banquet.

December 8, 1931: "Operational calculus" by Wayne Birchard and D. Paul Needham.

January 27, 1932: "Anharmonic ratios" by E. W. Carr.

March 9, 1932: "Harmonic analysers" by George Fink.

April 13, 1932: "Thermionic and Photo-electric work functions" by R. M. Bowie.

May 12, 1932: "The theory of relativity" by J. V. Atanasoff.

May 24, 1932: Initiation and banquet.

MARIANNE PREUSS, *Secretary*

LOCAL MATHEMATICS CLUBS

The Mathematics-Science Club of Drake University

Our club is a combination of Mathematics, Physics and Astronomy. The aim of the club is to give the students a chance to hear interesting topics reported on and discussed and also to promote friendship between its members.

Students having completed nine semester hours doing work on a major in one of the above mentioned subjects are eligible to membership. Selection is made by an unanimous vote of the members from those having the above qualifications. Grades are considered in the selection of

candidates but there is no specific requirement. At present we have thirty-one members including six faculty members.

The officers for 1931–1932 were: Ralph Collins, President; Guy Brunk, Vice President (first semester); Earling Jensen, Vice President (second semester); Mary Neff, Secretary-Treasurer: Martha Burton, Corresponding Secretary. The officers are elected at the April meeting.

The meetings and programs were as follows:

October 29, 1931: Picnic meeting. The names of prospective members were put before the members of the club.

November 10, 1931: "Einstein's theory of relativity" by Professor Helmick.

December 8, 1931: Initiation of six new members.

January 12, 1932: Moving picture reels showing the work in the General Electric laboratories at Schenectady were shown.

February 13, 1932: A special meeting called for the election of new members.

March 8, 1932: "Imaginaries—DeMoivre's theorem" by Professor Neff. Initiation of ten new members.

April 12, 1932: "Latest theories of the universe as presented by De Sitter and Einstein" by President Morehouse. Initiation of two new members.

May 10, 1932: Picnic meeting at the Observatory.

MARTHA ALICE BURTON, *Corresponding Secretary*

The Mathematics Club of the College of the City of Detroit

The officers for 1931–1932 were: John Malley, President; Claire G. Groteau, Secretary; Morris A. Greenberg, Treasurer. The officers are elected annually at the first meeting of the academic year.

The purpose of the society is to stimulate and promote an interest in mathematics.

The meetings and programs were as follows:

November 3, 1931: "An appreciation of Fourier's series" by Mr. Morris Rose.

December 1, 1931: "Method for approximating real roots of equations" by Dr. T. R. Running of the University of Michigan.

December 4, 1931: "Complex singularities" by Mr. Clarence Wylie.

January 20, 1932: "Magic squares" by Mr. Morris A. Greenberg.

March 17, 1932: "Some peculiar functions" by Mr. Morris Friedman.

April 26, 1932: "The trisection of the angle" by Miss Cecelia Maier.

May 10, 1932: "Some non-commutative algebras" by Mr. Edward Brushaber.

May 31, 1932: "The nine-point circle" by Miss Virginia Eyre.

CLAIRE G. CROTEAU, *Secretary*

The Napierian Club of DePauw University

The officers for 1931–1932 were: Kenneth Griffin, '32, President; Dorothy Wurst, '32, Vice President; Kenneth Smith, '32, Secretary; Harold Frey, '32, Treasurer.

The following programs were given at the regular monthly meetings:

September 28, 1931: Election of officers; election of new members.

October 21, 1931: Steak roast; election of other members.

November 5, 1931: "Life and work of Napier" by Kenneth Griffin; "Introduction to the trisection of an angle" by Frank Reid.

December 3, 1931: "Invention of calculus" by Anne Nichols; "Clippings on the trisection of an angle" by Kenneth Smith; "Japanese method of trisection" by Howard Stafford; "Report of mathematics convention" by Professor Hildebrandt.

January 7, 1932: "Magic squares" by Otto Behrens; "Pursuit problems" by W. E. Smith; "A determination of Avogadro's number by the Brownian movement" by Richard Humphreys.

February 4, 1932: Social Meeting. "Eminent American mathematicians" by Professor Edington; "Mathematical games" by Dorothy Wurst.

March 10, 1932: "Determination of Pi" by John Voliva; "Demonstration of the use of the calculating machine" by Professor Hildebrandt.
April 4, 1932: "Sixth degree curve fitting" by Professor Greenleaf.
May 5, 1932: Social meeting; election of new officers. "How to do research" by Arthur Noble; "Egyptian pyramids" by Harold Frey; "Trick problems" by W. E. Smith; "Problems" by Gordon Hiatt; "Mathematical spelling match" by Professor Arnold.

KENNETH SMITH, *Secretary*

PROBLEMS AND SOLUTIONS

EDITED BY B. F. FINKEL, OTTO DUNKEL, H. L. OLSON, AND WM. FITCH CHENEY, JR.

ELEMENTARY PROBLEMS

Send all communications about Elementary Problems and Solutions to Wm. Fitch Cheney, Jr. Dept. Box 35, Storrs, Connecticut.

The Department of Elementary Problems welcomes problems believed to be new, and demanding no tools beyond those ordinarily furnished in the first two years of college mathematics. Problems may be submitted unaccompanied by their solutions.

PROBLEMS FOR SOLUTION

E 11. *Proposed by Wm. R. Ransom, Tufts College.*

This problem was offered by Mr. Francis of Exeter at a meeting of the Association of Mathematics Teachers of New England in 1914.

Circumscribed about a circle is an isosceles trapezoid, $ABCD$, in which $DC<AB$, and $AD=BC$. Two perpendiculars are drawn; DG perpendicular to AB at G, and GH perpendicular to AD at H. Show that DA, DG and DH are the arithmetic, geometric, and harmonic means, respectively, between the pair of parallel sides AB and DC.

E 12. *Proposed by Wm. Fitch Cheney, Jr., Connecticut Agricultural College.*

Two coplanar right triangles, AOC and BOC, have the common hypotenuse OC. Using vector methods, express the vector OC in terms of the vectors OA and OB.

E 13. *Proposed by Wm. Fitch Cheney, Jr., Connecticut Agricultural College.*

It is required to find two factors which, together with their product, contain each of the nine digits from one to nine just once. Each of the several solutions should be found. (Zero must not appear in any solution.)

E 14. *Proposed by Wm. Fitch Cheney, Jr., Connecticut Agricultural College.*

At country fairs there occasionally appears a man with five discs, each four inches in diameter, and a table on which is painted a larger red circle. He challenges anyone to place the five discs so they completely cover the red circle on the first attempt. It appears that a correct solution lacks central symmetry.

How large is the maximum red circle which may be covered by the five discs, and how must they be placed?

E 15. *Proposed by Mrs. Pearl C. Miller, Washington University.*

Prove that if two external angle-bisectors of a scalene triangle are equal, then the sines of the three interior half-angles form a geometric progression. By external angle-bisector is here meant that segment of the line bisecting the exterior angles at a vertex of a triangle, intercepted between that vertex and the opposite side of the triangle.

ADVANCED PROBLEMS

Send all communications about Advanced Problems and Solutions to B. F. Finkel, Springfield, Mo. All manuscript should be typewritten, with double spacing, and with margins at least one inch wide.

Problems containing results believed to be new, or extensions of old results are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known textbooks or results found in readily accessible sources, will not be proposed as problems for solution in the Monthly. In so far as possible, however, the editors will be glad to assist members of the Association in the solution of such problems.

PROBLEMS FOR SOLUTION

3579. *Proposed by B. F. Kimball, Schenectady, N. Y.*

Given

$$F(x, a) = \sum_{s=-\infty}^{+\infty} \frac{\sin (x + sa)}{x + sa}$$

where s takes on all integral values including zero. Evaluate $F(x, a)$ for all real values of x and a. Discuss the discontinuities of the function.

3580. *Proposed by J. B. Reynolds, Lehigh University.*

If a body dropping vertically from an airplane reaches a velocity of 120 mi./hr., which is 99 percent of its terminal velocity, in 40 sec. what is the value of n on the assumption that the resistance of the air is directly proportional to the weight of the body and the nth power of its velocity?

3581. *Proposed by Nathan Altshiller-Court, University of Oklahoma.*

Let a, b, c, be the edges of a trihedral angle S-abc, and a', b', c', the perpendiculars at S to the planes bc, ca, ab. If u is the axis of a cone of revolution passing through a, b, c, the line u is also the axis of a cone of revolution tangent to the planes $a'b', b'c', c'a'$.

3582. *Proposed by Dewey C. Duncan, University of California.*

If α and β are positive integers and $\beta > 2$, then $2^\alpha + 1$ is never divisible by $2^\beta - 1$.

3583. *Proposed by H. Grossman, New York.*

If from any point on a circle, chords be drawn to all vertices of a regular inscribed $3n$-gon, the sum of the n longest chords is equal to the sum of the $2n$ shortest ones.

3584. *Proposed by W. E. Buker, Leetsdale, Pa.*

Find the rational values of x and y for which

$$x^3 + y^3 = 2^3 + 1^3.$$

See Dudeney: Canterbury Puzzles

3585. *Proposed by R. E. Gaines, University of Richmond.*

If a tangent at P_1 to the cardioid $\rho = a(1+\cos\theta)$ cuts the curve again at P_2 and P_3, and if the normal at P_1 cuts the curve again at P_4, P_5, P_6, then (1) $\rho_2\rho_3/\rho_1$ is constant. Also (2) $\rho_1+\rho_2+\rho_3=\rho_4+\rho_5+\rho_6$; (3) if the chord $P_2P_3 = 2\frac{1}{2}a$, it subtends a right angle at the origin.

3586. *Proposed by R. E. Gaines, University of Richmond.*

If while an ellipse is turned about in its plane it remains tangent to a fixed straight line at a fixed point, its foci trace a curve whose area is $2\pi a(a-b)$.

SOLUTIONS

3495. [1931, 340]. *Proposed by Mannis Charosh, New Utrecht High School, Brooklyn, N. Y.*

Prove that the $\phi(m)$ integers less than m and prime to it constitute an automorphic group, with respect to multiplication and reduction to least positive residue modulus m.

Prove that if each member of any group in (1) be multiplied by $r(r \neq m)$, a new group will be formed which will be automorphic and also isomorphic with the first group. The modulus in this case is rm.

Solution by J. D. Hill, Brown University.

Let S: $a_1, a_2, a_3, \cdots, a_{\phi(m)}$, denote the set of $\phi(m)$ integers less than m and prime to it. It will first be shown that the set S forms an Abelian group with respect to the given law of combination.

I. *The Combination Property.* Let a_i, a_j be two equal or unequal members of the set S. If their product be divided by m the positive remainder will be less than m and prime to it. Hence the product a_ia_j is congruent modulo m to some member of S.

II. *The Associative Law.* In view of the law of combination, this property is evidently satisfied.

III. *The Identical Element.* If the identical element, I, exists there must be a member x of S such that the congruence $a_ix \equiv a_i \bmod m$, is satisfied for every

element a_i of S. But this congruence is equivalent to $x \equiv 1 \bmod m$, and since S contains the element 1 it follows that $I = 1$.

IV. *The Inverse Element.* Let a_i be any member of S and consider the congruence $a_i x \equiv 1 \bmod m$. Since a_i is prime to m this congruence has one and only one solution, $x \equiv x_i \bmod m$, where x_i is a certain member of S. Hence the inverse of a_i is x_i.

V. *The Commutative Law.* As in II above, this property is evidently satisfied.

The initial assertion has therefore been established. It remains to show that there exists a rearrangement of S which is isomorphic with S. Such an arrangement results immediately upon replacing in order each member of S by its inverse. For out of the relation $a_i a_j \equiv a_h \bmod m$, easily follows $a_i^{-1} a_j^{-1} \equiv a_h^{-1} \bmod m$. Hence the group S is automorphic. Moreover, the automorphism set up in this way will reduce to the identical automorphism only if the integers m have the property that each element in S is its own inverse. But the result of Problem 3488[1] proves that this can happen when and only when $m = 1, 2, 3, 4, 6, 8, 12$, or 24. For the cases where $m = 1, 2, 3, 4, 6$, one readily shows that no automorphism other than the identity exists, whereas in the remaining cases non-identical automorphisms do exist. For example, we have

$$\text{for } m = 8, \begin{pmatrix} 1 & 3 & 5 & 7 \\ 1 & 5 & 7 & 3 \end{pmatrix}$$

$$\text{for } m = 12, \begin{pmatrix} 1 & 5 & 7 & 11 \\ 1 & 7 & 11 & 5 \end{pmatrix}$$

$$\text{for } m = 24, \begin{pmatrix} 1 & 5 & 7 & 11 & 13 & 17 & 19 & 23 \\ 1 & 11 & 13 & 23 & 17 & 19 & 5 & 7 \end{pmatrix}$$

The statement we are asked to prove in the second part of the problem is true only if the condition $r \neq m$ is replaced by the condition r prime to m. For if the set S': $ra_1, ra_2, \cdots, ra_{\phi(m)}$ constitutes a group mod mr, we must have, for each a_i and a_j, an a_k such that $ra_i ra_j \equiv ra_k \bmod mr$, or $ra_i a_j \equiv a_k \bmod m$. But since a_k is prime to m, this necessitates that r be prime to m.

With r prime to m, $ra_i \equiv x_i \bmod m$, where the x's are exactly the a's in some order. If $x_i x_j \equiv x_s \bmod m$, we have $ra_i ra_j \equiv ra_s \bmod m$; and since obviously $ra_i ra_j \equiv ra_s \bmod r$, and r is prime to m, we have $ra_i ra_j \equiv ra_s \bmod mr$. Hence the correspondence $ra_i \sim x_i$ between the elements of S' and S establishes an isomorphism.

3518. [1931, 588]. *Proposed by H. A. Simmons, Northwestern University.*

In Bôcher's *Higher Algebra*, p. 11, Exercise 2, the student is expected to make a statement which may be expressed as follows: If f is a polynomial in m variables $x_1, x_2, \cdots, x_m$ which is known not to be of higher degree than n_i in x_i, $(i = 1, \cdots, m)$, and if f vanishes at the $(n_1+1)(n_2+1) \cdots (n_m+1)$ distinct points

[1] See this MONTHLY, vol. 39 (1932), p. 241.

(1) $$[x_1^{(i_1)}, x_2^{(i_2)}, \cdots, x_m^{(i_m)}],\ i_j = 1, 2, \cdots, n_j + 1;\ j = 1, \cdots, m,$$

then f vanishes identically.

We ask the question: What is the greatest number of the points (1) at which a polynomial f of the type hypothesized can vanish and yet not be identically zero?

Solution by Jeanette Fox, Student, University of Berlin.

Let

(1) $$x_i^{(1)}, x_i^{(2)}, \cdots, x_i^{(n_i+1)},$$

be n_i+1 distinct given values of the independent variable x_i; set

(2) $$g_i(x_i) = [x_i - x_i^{(1)}][x_i - x_i^{(2)}] \cdots [x_i - x_i^{(n_i+1)}];$$

and denote by $g_i'(x_i)$ its derivative with respect to x_i. Consider the polynomial of degree n_i in x_i

(3) $$\frac{g_i(x_i)}{g_i'[x_i^{(1)}][x_i - x_i^{(1)}]}.$$

For $x_i = x_i^{(1)}$, (3) has the value unity, whereas it is zero for any other value of x_i in the set (1). Hence the polynomial

(4) $$f(x_1, x_2, \cdots, x_m) = \frac{g_1(x_1)g_2(x_2)\cdots g_m(x_m)}{g_1'(x_1')g_2'(x_2')\cdots g_m'(x_m')(x_1 - x_1')(x_2 - x_2')\cdots(x_m - x_m')}$$

(where each x_i' is equal to any root of $g_i(x_i)$) has the value unity at $(x_1', x_2' \cdots x_m')$ and the value zero at any one of the remaining sets of points $[x_1^{(i_1)}, x_2^{(i_2)}, \cdots, x_m^{(i_m)}]$. Thus the polynomial defined by (4) is certainly not identically zero; it is of degree n_i in x_i; and it vanishes at

(5) $$(n_1 + 1)(n_2 + 1)\cdots(n_m + 1) - 1$$

points. Hence (5) is the maximum number of points at which a polynomial of the described degree type can vanish without being identically zero.

NEWS AND NOTICES

Readers are invited to contribute to the general interest of this department by sending items to Professor J. H. Weaver, Ohio State University, Columbus, Ohio.

The Institute for Advanced Study will begin active work in the autumn of 1933. It will be located near Princeton, New Jersey, and will consist of a series of schools, the first of which will be the school of mathematics. Through the courtesy of the authorities of Princeton University, the mathematics group will be temporarily accommodated at the new Fine Hall, which is peculiarly adapted to the purposes of an institute. Professor Albert Einstein of Berlin was appointed professor of mathematical and theoretical physics at the institute. Professor Oswald Veblen was also appointed a professor at the School of Mathematics. Dr. Walter Mayer of Berlin was made associate in mathematics and Dr. J. L. Vanderslice was appointed assistant to Professor Veblen.

The University of Oregon and Oregon State Agricultural College have been put under one board and one chancellor, and a number of changes in function have been made. One of these is to place all advanced work in science and mathematics at the State College, and Professor W. E. Milne, who has been professor of mathematics at the University since 1919, has been made head of the department of mathematics at the State College.

Dr. Jacques Hadamard, professor of mathematics at the College de France and a member of the Institute de France, plans to visit the United States in the spring of 1933.

Dr. Jan H. Oort, lecturer and observer at the University of Leyden, arrived in Delaware, Ohio, October 4, to begin photometric studies of nebulae in a co-operative plan with the Perkins Observatory of Ohio Wesleyan University. While at the Perkins Observatory Dr. Oort will give a special series of lectures on stellar statistics on Thursday afternoons, beginning October 13. The titles of the lectures are: "The Observational Data," "Methods of Determining Stellar Distributions," "General Structure of the System of Stars," "Stellar Dyanmics I," "Stellar Dynamics II," and "Galactic Rotation." The lectures and discussions are open to all interested.

Professor H. Weyl of the University of Goettingen has accepted the invitation of Swarthmore College to give the William J. Cooper Foundation Lectures for 1932–33. He will reside at Swarthmore from the middle of February until the middle of March. During this period he will give five lectures on the general subject of Mind and Nature and will take part in the honors work of the college. The lectures are open to the public.

Dr. Alois Kovarik, professor of physics at Yale University, was the recipient on July 2 of an honorary Dr. of Science from Charles University in Prague and on June 10 of the Memorial Medal from Comenius University at Bratislavia.

Dr. Leonard Carlitz has been appointed assistant professor of mathematics at Duke University.

Dr. M. R. Hestenes has been appointed research assistant at the University of Chicago.

Associate Professor V. B. Hinsch has been promoted to a professorship of mathematics at the Missouri School of Mines and Metallurgy.

Dr. V. A. Hoersch has been promoted to the rank of associate in mathematics at the University of Illinois.

E. D. McCarthy has been appointed assistant professor of mathematics at the University of Detroit (Engineering College).

Dr. David Moskovitz, of Brown University, has been appointed assistant professor of mathematics at the Carnegie Institute of Technology.

Dr. F. C. Ogg has been appointed associate professor of mathematics at Bowling Green State College, Bowling Green, Ohio.

Dr. Mina S. Rees has been promoted to an assistant professorship of mathematics at Hunter College.

Professor B. D. Roberts, of Parsons College, has been appointed professor and head of the department of mathematics at New Mexico Normal University.

Dr. Otto Struve, assistant director at the Yerkes Observatory, has been appointed director of the observatory and professor of astrophysics at the University of Chicago.

Dr. Norbert Wiener has been promoted to a professorship of mathematics at the Massachusetts Institute of Technology.

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CORRIGENDA

Volume XXXIX 1932:

P. 28, A Correction to vol. XXXV, p. 11.
P. 487, line 13, for "V. H. Ivanoff" read "V. F. Ivanoff."
P. 506, line 13, for "R. M. Ashmun" read "R. N. Ashmun."
P. 507, line 9, add: "by invitation;" and, line 11, omit "by invitation."
P. 510, line 4, for "Mathias" read "Mathis."
P. 537, fifth line from bottom, for "Aitkin" read "Aitken."
P. 547, sixth line from bottom, for "1625" read "1525."
P. 555, line 15, for "Smith" read "Smyth."

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MEETINGS OF THE ASSOCIATION AND ITS SECTIONS

Seventeenth Annual Meeting of the Association, Atlantic City, N.J., Dec. 27-30, 1932.

The following is a list of the sections of the Association, with dates of those section meetings which have been scheduled for 1932 and reported to the Secretary.

ILLINOIS, Urbana, May 6-7.
INDIANA, Indianapolis, May 6-7.
IOWA, Cedar Falls, April 29-30.
KANSAS, Topeka, Feb. 13.
KENTUCKY, Lexington, May.
LOUISIANA-MISSISSIPPI, Oxford, Miss., March 11-12.
MARYLAND-DISTRICT OF COLUMBIA-VIRGINIA, Baltimore, Md., Dec. 3.
MICHIGAN, Ann Arbor, March 19.
MINNESOTA, River Falls, Wis., May 7.
MISSOURI.
NEBRASKA, Omaha, May 6-7.
OHIO, Columbus, Ohio, April 7.
PHILADELPHIA, Philadelphia, Pa., Nov. 26.
ROCKY MOUNTAIN, Laramie, Wyo., April 15-16.
SOUTHEASTERN, Gainesville, Fla., Mar. 18-19.
SOUTHERN CALIFORNIA, San Diego, March 26.
TEXAS, Austin, Jan. 30.

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DEC 27 1932

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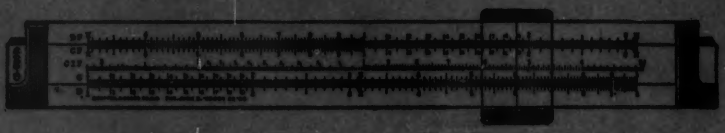

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Numerous and varied exercises have been included. Some of them make no demand on algebraic skill but were designed to develop insight and a real understanding. Others illustrate interesting applications of trigonometry to different fields or provide facility in the use of trigonometric relations. Continuous review is planned. A complete chapter on logarithms is included, and the results of logarithmic computation are given as worked from four- and five-place tables.

Available with or without the MACMILLAN LOGARITHMIC AND TRIGONOMETRIC TABLES, *edited by E. R. Hedrick.*

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